Multifactor mathematical modelling of ecological and economic systems (the example of coal mining development)

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SUMMARY

The model for planning and forecasting of coal resources as a component of the management system of the national raw material base has been developed as a result of the study.

The model of national potential for coking coal’s rational use management based on balancing flows of coal extraction, coke and semi-coke production according to the needs of the country's metallurgical industry (ironmaking) has been proposed. The model is confirmed by the official statistics.

The model of anthracite’s and other coals’ rational extraction management has been proposed based on balancing flows of coal extraction, its production according to the energy sector needs (heat production). The model is confirmed by the official statistics.

In order to solve the problem of system management of national coal mining, it has been proposed to balance the volumes of coal mining with the volumes of its consumption in metallurgy and energy sectors. The necessity of planning and forecasting the extraction volumes has been proved taking into account cyclical nature of economic processes’ development. It has been proposed to use a multi-factor economic and mathematical model for forecasting the development of complex systems as a sample frame.
Introduction
Ukraine occupies one of the leading places in the world in terms of mineral resources per capita and their production. It mines up to 4% of the world mineral raw materials each year. Taking into account the importance of the resource provision for the country's development, as well as of the mineral and raw material base (MMB) reproduction and the creation of conditions for the growth of minerals, National Program for the Development of MMB in Ukraine up to 2030 was adopted. Difficult economic situation in the country and significant reduction in budget funding of exploration made impossible to achieve the expected rates of MMB reproduction by 2017. To ensure program’s implementation, it is important to find the ways to increase the effectiveness of Ukrainian MMB exploitation and build the system of minerals’ balanced mining, production, export and import management.

Method
In the analysis, general-scientific methods (analysis and synthesis, induction and deduction) and special methods of phenomena and processes analysis (abstraction, econometric and econometric-mathematical modeling) have been used.

Results
Ukraine has significant mineral resources of non-metallic minerals, which can be used in various branches of building materials industry due to their composition and technological properties: cement, wall materials of natural stones, non-metallic rubble-stone and facing materials, porous fillers, glass, brick-tiling, building ceramics, etc. The study is devoted to non-metallic minerals for construction, namely: building stone, construction sand and expanded clay.

Findings’ analysis revealed that the dependencies reach their peak in 2006-2007s, 2011-2012s; minimum in 2009 and 2014-2015s (fig.1). Moreover, fluctuations of studied indices of non-metallic mineral resources extraction for construction needs have periodic, cyclic nature. We consider that these fluctuations reflect short cycles of economic development lasting 4-5 years. The comparisons of the obtained results and overall economic situation in the country show that maximum of volumes of extraction and the growth rates coincide with the years when Ukraine's economy grew.

\[
y = -0.0002x^6 + 1.9373x^5 - 9982.1x^4 + 3E+07x^3 - 4E+10x^2 + 3E+13x - 1E+16
\]
\[R^2 = 0.8183\] (expanded clay)

\[
y = 0.0037x^6 - 44.887x^5 + 225377x^4 - 6E+08x^3 + 9E+11x^2 - 7E+14x + 2E+17
\]
\[R^2 = 0.8674\] (building stone)

\[
y = -0.0061x^6 + 73.088x^5 - 367524x^4 + 1E+09x^3 - 1E+12x^2 + 1E+15x - 4E+17
\]
\[R^2 = 0.7945\] (construction sand)

![Figure 1 Growth rates of building stone, construction sand and expanded clay mining during 2004–2016 and the appropriate approximation curves](image-url)

It is possible to simulate the situation for each individual mineral, without taking into account interconnections applying traditional methods of approximation and smoothing. Therefore, in order to
predict the development of the industry, it has been proposed to use the author's multifactorial economic and mathematical model for forecasting the development of complex systems.

To describe interdependent processes of mineral resources extraction’s business cycles’ development taking into account interconnections between them, the system of differential equations is proposed:

\[
\begin{align*}
\frac{dN_1}{dt} &= N_1(\varepsilon_1 + \gamma_1 N_2), \\
\frac{dN_2}{dt} &= N_2(\varepsilon_2 + \gamma_2 N_1), \\
\frac{dN_i}{dt} &= N_i(\varepsilon_i + \gamma_i N_{i-1})
\end{align*}
\]  

(1)

where \(i\) – number of subsystems in a complex system; \(N\) – volumes of mineral resources extraction; \(\varepsilon\) – increment rate of volumes of mineral resources extraction, if there is no correlation with other volumes (it is constant of proportionality which states growth rates of volumes of extraction \(\frac{dN}{dt}\) to \(N\) ratio), \(\gamma\) - increment rate of need in mineral resources.

The system (1) is comprised of three equations (number of resources) as three kinds of mineral resources are selected for analysis, alike: building stone, mason sand and expanded clay. The further analysis and forecasting will be based on the following indices: base growth rate \(n\) and average of volumes of extraction \(K\). The correlation is: \(n = \frac{N}{K}, K = \frac{\varepsilon}{\gamma}, \varepsilon = \frac{\ln N}{N_0 - N_0 t}\).

Figure 2 shows the obtained dependencies of the growth rates of construction sand \(\varepsilon_1(t)\), expanded clay \(\varepsilon_1'(t)\) and building stone \(\varepsilon_2(t)\) mining. Dependences of the growth rates \(\varepsilon_i(t), \varepsilon_i'(t)\) and \(\varepsilon_2(t)\) characterize the dynamics of each individual mineral mining independently.

The curves of the obtained growth rates indexes of the named minerals demand have been built:

\[
\gamma_1 = \frac{m_1 \varepsilon_1}{N_1}, \quad \gamma_1' = \frac{m_1 \varepsilon_1'}{N_1}, \quad \gamma_2 = \frac{m_2 \varepsilon_1}{N_2} \quad \text{and} \quad \gamma_2' = \frac{m_2 \varepsilon_1'}{N_2}
\]

Dynamics of dependencies for the coefficients \(\gamma(t), \gamma'(t), \gamma(t)\) and \(\gamma'(t)\) presented in Fig. 3 shows that they correlate with each other on time. This is quite logical, since when building the model it was considered that the coefficients \(\gamma, \gamma', \gamma\) and \(\gamma'\) are values that reflect the needs of construction for sand extraction, the need of expanded clay production for expanded clay mining, the need for stone extraction depending on the needs for the production of sand and expanded clay raw materials respectively. These values should be fully correlated, since during the studied period there was no change in the production technology.
The system of equations (1) reveals the dependence of the growth rate of the expanded clay extraction $\lambda_1'(t) = \varepsilon_1(t) + \gamma_1(t) N_2(t)$, the growth rate of building stone mining depending on the amount of expanded clay extraction $\lambda_2'(t) = \varepsilon_2(t) + \gamma_2(t) N_1(t)$, the growth rate of construction sand production $\lambda_1(t) = \varepsilon_1(t) + \gamma_1(t) N_2(t)$, the growth rate of construction stone mining depending on the volumes of construction sand production $\lambda_2(t) = \varepsilon_2(t) + \gamma_2(t) N_1(t)$, if there is a correlation between them (fig. 4).

Correlations $\lambda_1'(t) = \varepsilon_1(t) + \gamma_1(t) N_2(t)$, $\lambda_2'(t) = \varepsilon_2(t) + \gamma_2(t) N_1(t)$, $\lambda_1(t) = \varepsilon_1(t) + \gamma_1(t) N_2(t)$, $\lambda_2(t) = \varepsilon_2(t) + \gamma_2(t) N_1(t)$ (fig. 4) correlate with each other on time to a greater extent than the corresponding coefficients $\varepsilon(t)$, $\varepsilon_1(t)$, $\varepsilon_2(t)$ and $\gamma(t)$ (fig. 2). This is due to the fact that growth rates’ time changes shown in Fig. 4 are based on interconnections between three minerals extraction.

It is proved that such a model will allow controlling the volumes of mining of technologically related minerals in interdependence on the volumes of their use.

The work develops a systematic approach to the management of mining, using, export and import non-metallic minerals for construction considering the economic parameters of the development of the world market situation and the domestic market as well.

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**Figure 3** Dynamics of the demand rate in construction sand, expanded clay and building stone mining

**Figure 4** Dynamics of the growth rates of construction sand, expanded clay and building stone mining if there is a correlation between them
The analysis showed that the classification of minerals used by the Public Service Geology and Mineral Resources of Ukraine, the State Statistics Committee of Ukraine and Statistics of foreign economic activity - codes UKTZED - do not coincide. This complicates the study process using these official sources.

According to the results of the conducted researches, the indicators of the dynamics of minerals production in their natural and costly dimensions have been found to be inadequate. It is proved, that total import and export volumes decreased during 2011 – 2017s. Natural sand export prices dropped from 27,81 USD per Ton in 2011 to 18,91 USD per Ton in 2017. Import prices decreased as well from 423,02 USD per Ton in 2011 to 248,78 USD per Ton in 2017. We have found out key differences in export-import price scan. We think that is because the code of 2505 item includes a few kinds of sand, which prices significantly differ. It is possible to predict that high import prices are caused by the fact of mason sand export (cheap) and quartz sand import (more expensive). The obtained results prove that share of natural sand exports is 3-5% of the volume of its extraction in the country.

Pebble stone, gravel and crushed stone export prices fell from 13,582 USD per Ton in 2011 to 5,783 USD per Ton in 2016. Export prices grew insufficiently to 6,027 USD per Ton in 2017. Import prices rose during 2011 – 2013s to 90,437 USD per Ton in 2013. During 2014 – 2017 prices declined to 58,383 USD per Ton in 2017. The comparison of pebble stone, gravel, crushed stone export and import prices show that import prices change is insignificant, while exports decreased almost twice. Besides, it should be noted that export prices are almost 10 times lower than of import. It is determined that the share of pebble stone, gravel, crushed stone export is 20-30% of the volume of their extraction in the country. Thus, significant part of the resources is exported in the context of negative external market price situation (high import prices and low export prices). Foreign and domestic market price comparison shows that alike export prices decline during 2013-2015s, domestic prices of stone, sand and clay producers increased substantially (almost 20%). This approach to the formation of export flows of mineral resources we consider unacceptable, since it poses threat to the financial situation in mining industry and the country as a whole.

Analysis of the geographical structure of natural sand, pebbles, gravel and crush stone export and import allowed proposing a systematic approach to rational extraction, the use of the country's mineral resources base, while taking into account export-import flows and the price conditions of the external and internal markets and taking into account the resource component of state security. The obtained results indicate that the domestic export of the analyzed mineral resources for construction is not sufficiently cost-effective, since the price for exported resources is too low. Moreover, it should be pointed out that there is a real need for pebble stone, gravel and crushed stone import to meet their own needs in the Republic of Belarus, and the Russian Federation has its own substantial reserves of these minerals. We believe that the position of the Russian Federation is explained by the attempt to use other countries' mineral resources and to save its own. A similar situation regarding raw materials is known in steelmaking industry. For over 20 years, the world steelmaking leaders have preferred to import raw materials, namely ore and metallurgical coal, not exploiting their own deposits.

**Conclusion**

Taking into account the fact that the implementation of the National Program for the Development of Ukrainian raw material base by 2030 is under threat as a result of the permanent lack of funding for exploration works, the funds received from export operations can be used for its financing. However, one should pay attention to the fact that the main tasks of the program are to reproduce and increase the reserves of mineral resources. It is possible to solve these important tasks not only conducting geological prospecting work, but above all by creating conditions for more rational extraction and exploitation of the country’s mineral raw material base. In our opinion, resources exhaustibility and ineffective export operations, endanger the resource component of national security.