Modeling of pushing processes in anisotropic low permeable Gas reservoirs

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SUMMARY

On the base of combined finite element - difference method, we carried out computer modeling of filtration processes between producing and injection wells in anisotropic low permeable gas reservoirs. The modeling results show that intensity of the filtration processes between producing and injection wells and, respectively the gas production level depends essentially on their location, as in the shear-isotropic so in anisotropic gas reservoirs. Accordingly obtained information for effective exploitation of low permeable shear-isotropic gas reservoirs it is necessary to place the production and injection wells along the major axis of the reservoir anisotropy. In the case of production and injection wells installation in the low permeable anisotropic gas reservoirs, the most effective gas exchange between them will take place when the direction of the increased permeability of the reservoir coincides with the direction of the wells location. Obviously, we can achieve the best conditions for gas production in anisotropic low permeable gas reservoirs only after systematic analysis and optimal selection of the most important factors of the filtration process in every practical case.
Introduction

In our days, there are remaining problems of growth efficiency of the low permeable anisotropic gas reservoirs exploitation (Ter-Sarcisov, 1999), (Yaskin, et. al., 2018). These problems mainly connected with increasing of the gas recovery and achievements of economically effective ways of gas reservoirs exploitation. In this situation, computer methods of anisotropic gas-condensate reservoirs modeling are very powerful, because they allow getting and analyzing information in vicinity of gas production and gas injection wells, which is necessary for effective supporting of gas-condensate raw producing. In addition, they allow detection and evaluation of specific filtration fiches and uncertainties, which appear due to limited geologic information outside the wells. All such information we can obtain by comparatively cheap way and use for effective installation of system gas production and injection wells in anisotropic heterogeneous reservoirs. At that time, there are many computer methods, which allow resolving of different practical gas-condensate production modeling problems (Aziz, 2004), (Chen, et. al., 2006), (Ertekin, et. al., 2001). Another hand, in our time there remain some problems, which connected with accuracy and adequacy of anisotropic heterogeneous gas-condensate reservoirs modeling in real conditions of low permeable gas reservoirs exploitation. Especially important remains problem of effective controlling of anisotropic filtration process (possibly with the help of recent technologies (Yaskin, et. al., 2018)) between producing and injection wells with purpose of effective gas production supporting. Presented in this work, combined finite element-differences method of resolving nonstationary anisotropic piezoconductivity Lebenson problem, with calculation of heterogeneous filtration parameters in gas-condensate reservoir and gas penetration its boundaries, has a good convergence and steadiness of problem resolving. So it allows adequately calculate gas reservoir pressure distribution in real conditions of gas production in anisotropic low permeable reservoirs and has some advantages in comparison with the same methods.

Mathematical formulation and solving problem

We will consider productive gas-condensate reservoirs in which we can neglect by presence of liquid phase. We also suggest that average thickness of productive gas reservoir considerably smaller than its horizontal sizes. At that case, we can use the ordinary formulation of the plane nonstationary anisotropic piezoconductivity Lebenson problem in rectangular axes (X, Y) such as (Lubkov, 2019):

\[
\frac{\partial P^2}{\partial t} = \frac{1}{c} \left( k_{xx} \frac{\partial^2 P^2}{\partial x^2} + k_{yy} \frac{\partial^2 P^2}{\partial y^2} + 2k_{xy} \frac{\partial P^2}{\partial x} \frac{\partial P^2}{\partial y} \right) + \gamma; \\
P(t = 0) = P_0; \\
k_{sgr}gradP^2 = \alpha(P^2 - P_b^2). 
\]

Here (1) – nonstationary anisotropic piezoconductivity Lebenson equation; (2) – initial condition; (3) – condition of gas infiltration in reservoir’s borders; P(x,y,t) – pressure, as function of coordinates and time; c = η/m - coefficient of Lebenson piezoresistivity; k_{xx},k_{yy},k_{xy} - anisotropic coefficients of gas permeability; η – gas dynamic viscosity; m – porosity of gas reservoir; P_0 – initial pressure of the gas reservoir; P_b – gas pressure in the border of investigating area; k_s – gas boundary permeability; α - coefficient of the gas infiltration in the border of reservoir; γ - power gas well production or injection parameter. For resolving nonstationary anisotropic piezoconductivity Lebenson problem, we use variation finite element method, which leads to the solving of variation Lebenson piezoconductivity equation:

\[
\delta I(P) = 0. 
\]

Here I(P) – functional of anisotropic piezoconductivity Lebenson problem (1) – (3), which after substitution \( \tilde{P} = P^2 \) can be presented as functional of usual anisotropic piezoconductivity problem (Lubkov, 2019):

\[
I(\tilde{P}) = \frac{1}{2} \int_S \left\{ k_{xx} \left( \frac{\partial \tilde{P}}{\partial x} \right)^2 + k_{yy} \left( \frac{\partial \tilde{P}}{\partial y} \right)^2 + 2k_{xy} \frac{\partial \tilde{P}}{\partial x} \frac{\partial \tilde{P}}{\partial y} \right\} + 2 \int_L c \frac{\partial \tilde{P}}{\partial t} d\tilde{P} - 2\gamma \tilde{P} d\tilde{P} - 1 \int_L \alpha(\tilde{P} - 2\tilde{P}_b) \tilde{P} dl 
\]

here S – the square of investigating area; L – contour, which surrounds the square S, dl – element of the contour.
For resolving variation equation (4) we use eight-nodal isoparametric quadrangular finite element (Lubkov, 2019). As global coordinate system, where we unit all finite elements of investigating area S, rectangular system (X, Y) is using. As local coordinate system, where in limits of every finite element we define approximation functions \( \phi_i \) and make numerical integration, normalizing coordinate system (\( \xi, \eta \)) is used. In that system coordinates, pressure, initial pressure, pressure in the border of investigating area, coefficient of gas penetration in the reservoir border and derivatives of pressure on coordinates approximated in such way:

\[
x = \sum_{i=1}^{8} x_i \phi_i; y = \sum_{i=1}^{8} y_i \phi_i; P = \sum_{i=1}^{8} P_i \phi_i; \tilde{P}_0 = \sum_{i=1}^{8} P_{i,0} \phi_i; \vec{P} = \sum_{i=1}^{8} P_{b,i} \phi_i; \alpha^2 = \sum_{i=1}^{8} \alpha_i \phi_i;
\]

\[
\frac{\partial \tilde{P}}{\partial x} = \sum_{i=1}^{8} P_i \frac{\partial \Psi_i}{\partial y} = \sum_{i=1}^{8} P_i \frac{\partial \Psi_i}{\partial y} = \frac{1}{\left| J \right|} \left( \frac{\partial \phi_i}{\partial \eta} \frac{\partial \phi_i}{\partial \xi} - \frac{\partial \phi_i}{\partial \xi} \frac{\partial \phi_i}{\partial \eta} \right); \Phi_i = \frac{1}{\left| J \right|} \left( \frac{\partial \phi_i}{\partial \xi} \frac{\partial \phi_i}{\partial \eta} - \frac{\partial \phi_i}{\partial \eta} \frac{\partial \phi_i}{\partial \xi} \right); \]

(6)

Here \( J = \frac{\partial y}{\partial \eta} \frac{\partial x}{\partial \xi} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta} \) - Jacobian matrix between systems \((x, y)\) and \((\xi, \eta)\).

Following to variation equation (4) and suggesting, that nodal meanings from derivatives of pressure on time \( \frac{dP_k}{dt} \) known values and cannot be variated, we get system of differential equations for \( k \) – nodal of \( p \) – finite element in such view:

\[
\frac{\partial I_p}{\partial P_k} = \sum_{i=1}^{8} \{ H_{ki}^p \frac{dP_i}{dt} + (A_{ki}^p + Q_{ki}^p) P_i - Q_{ki}^p \} - \gamma_k^p = 0;
\]

(7)

\[
H_{ij}^p = \int_{-1}^{1} \int_{-1}^{1} c_i \phi_j \left| \partial \Psi \right| d\xi d\eta; A_{ij}^p = \int_{-1}^{1} \int_{-1}^{1} \left( k_{xx} \Psi_i \phi_j + k_{xy} \phi_i \phi_j + k_{yy} \Psi_i \Psi_j \right) \left| \partial \Psi \right| d\xi d\eta; Q_{ij}^p = \int \alpha \phi_i \phi_j d\xi d\eta;
\]

\[
\gamma_k^p = \int_{-1}^{1} \int_{-1}^{1} \phi_i \left| \partial \Psi \right| d\xi d\eta.
\]

For resolving the system of linear differential equations of the first order (7) at initial conditions (6) we use method of finite differences. At that, approximation of derivative in time we can realize on the base of implicit differential scheme (Aziz, 2004):

\[
\frac{d\tilde{P}}{dt} = \tilde{P}(t + \Delta t) - \tilde{P}(t).
\]

(8)

Putting expression (8) into the system (7), we obtain the next system of linear algebraic equations:

\[
\sum_{i=1}^{8} \left( \{ H_{ki}^p + A_{ki}^p + Q_{ki}^p \} P_i(t + \Delta t) - \frac{1}{\Delta t} H_{ki}^p \frac{P_i(t) - Q_{ki}^p}{\Delta t} \right) - \gamma_k^p = 0 \quad (k = 1-8).
\]

(9)

After summing equations (9) at all finite elements, we obtain the global system of linear algebraic equations, which allows defining unknown meanings of gas pressure at the moment of time \( t + \Delta t \) via their meanings at previous moment \( t \). Further, we resolve the global system equations on the base of Gauss numerical method (Lubkov, 2019). Because of solving, we can define pressure in all nods of the finite element net. Therefore, we can determine reservoir gas pressure in any points of investigating reservoir area in any times.

**Modeling of anisotropic gas pushing processes**

In modeling, we consider filtration processes between producing and injection wells in anisotropic low permeable gas reservoir. Let’s suggest that power of producing and injection wells are 24840 m³ over day at reservoir pressure 10 MPa. With calculation of gas expansion when going out, the gas well power will be 2,484⋅10⁶ m³ over day. Let us suggest that area of considering gas reservoir is 9×9 km². We choose some characteristic parameters of the gas reservoir: \( \eta = 0,18⋅10^{-4} \text{ Pa}\cdot\text{s}; m = 0,15; P_0 = 1\text{ MPa}, \) at that coefficient of gas piezoresistivity \( c = 0,27⋅10^{-12} \) (Basniev, et. al, 2003). We suggest not gas penetrating processes in the boundaries of the gas reservoir.
We have presented obtained results of gas pushing processes modeling in the figures 1 – 4:

**Figure 1.** a, b Distribution of pressure between production and injection wells over month from beginning in shifting-isotropic case of permeability gas distribution: a) \( k_{xx} = 0.0012D, k_{yy} = 0.0012D, k_{xy} = 0, 0012D \); b) \( k_{xx} = 0, 0012D, k_{yy} = 0, 0012D \). (1D (Darzi) = 10^{-12} m^2).

**Figure 2.** a, b Distribution of pressure between production and injection wells at their horizontal installation relatively of the main anisotropy axes over month from the action beginning: a) \( k_{xx} = 0, 012D, k_{yy} = 0, 0012D, k_{xy} = 0, 0012D \); b) \( k_{xx} = 0, 0012D, k_{yy} = 0, 012D, k_{xy} = 0, 0012D \).

**Figure 3.** a, b Distribution of pressure between production and injection wells at their vertical installation relatively of the main anisotropy axes over month from the action beginning: a) \( k_{xx} = 0, 012D, k_{yy} = 0, 0012D, k_{xy} = 0, 0012D \); b) \( k_{xx} = 0, 0012D, k_{yy} = 0, 012D, k_{xy} = 0, 0012D \).
Figure 4. a, b Distribution of pressure between production and injection wells at their diagonal (shifting) installation relatively of the main anisotropy axes over month from the action beginning: a) \(k_{xx} = 0.012D, k_{yy} = 0.0012D, k_{xy} = 0.0012D\); b) \(k_{xx} = 0.0012D, k_{yy} = 0.012D, k_{xy} = 0, 0012D, k_{yy} = 0, 012D\).

Conclusions

The elaborated combined finite element - difference method of resolving nonstationary anisotropic piezoconductivity Lebenson problem in heterogeneous gas reservoirs allows us adequately in quantitative level to describe the pressure distribution in the low permeable gas reservoirs between producing and injection wells. The modeling results show that intensity of the filtration processes between producing and injection wells and, respectively gas production depends essentially on their location, as in the shear-isotropic so in anisotropic gas reservoirs. Accordingly obtained information for effective exploitation of low permeable shear-isotropic gas reservoirs it is necessary to place the production and injection wells along the major axis of the reservoir anisotropy. In the case of production and injection wells installation in the low permeable anisotropic gas reservoirs, the most effective gas phase exchange between them will take place when the direction of the increased permeability of the reservoir coincides with the direction of the wells location. Obviously, we can achieve the best conditions for gas production in anisotropic low permeable gas reservoirs only after systematic analysis and optimal selection of the most important factors of the filtration process in every practical case. On the other hand, we can estimate all these factors with the help of modeling based on the presented method. In the future, it is interesting to create a practically significant method of optimization of gas production in the real conditions of anisotropic low permeable gas reservoir exploitation on the base of the developed combined finite element-difference method.

References

Ertekin, T. et. al. [2001] Basic applied reservoir simulation. Richardson, Texas.