Simulation of Water Filtration in a Geothermal Doublet


SUMMARY

The paper is devoted to modeling an open geothermal system consisting of two wells (production and injection). Geothermal energy is a promising type of renewable energy. A mathematical model of the functioning of such a system is described and parallel algorithms are developed for solving the problem of the distribution of cold water from an injection well in a geothermal reservoir. The results of numerical calculations and analysis of the efficiency of using parallel algorithms are presented.
**Introduction**

Now, the geothermal systems (Alkhasov, 2008; Bertani 2007; Ziagos et. al., 2013) as a rule, serves for the thermal energy delivering from the bowels of the earth (Aleksenko et. al., 2016; Gnatus, Khutorskoy 2010; Gnatus, Khutorskii, Khmelevsloy, 2011; Holm et al. 2010) and as a whole is a production facility that includes a reservoir (aquifer) of geothermal waters throughout the whole service life, that is a geological component with changing characteristics during operation (Vaganova, Filimonov, 2018; Vaganova, 2018).

**Mathematical model**

Let consider a mathematical model of an open geothermal loop system (Vaganova, Filimonov, 2015; Vaganova, Filimonov, 2016; Vaganova, Filimonov, 2016a), consisting of two perfect wells: the injection well $\Omega_1$ with the water temperature $T_1$, and the production well $\Omega_2$, with the water temperature $T_2(t) (T_1 < T_2(0))$, $\Omega$ is the computational areas (Fig.1a).

**Figure 1.** A basic scheme of simulated area (a).Temperature field in 15th year of exploitation in horizontal slice of the aquifer with the system of two wells (b).

In a general case, fluid motion in a geothermal aquifer $\Omega$ is described by the Navier-Stokes equations. For the equations of underground hydrodynamics describing the motion of water in a porous soil it is assumed that the fluid velocity components $V=(u,v,w)$ and the derivatives with respect to the coordinates $x, y, z$ are small. These equations are reduced to the system

$$
\begin{align*}
\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} &= \frac{g \sigma u}{k} , \\
\frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial y} &= \frac{g \sigma v}{k} , \\
\frac{\partial w}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial z} &= \frac{g \sigma w}{k} - g ,
\end{align*}
$$

(1)

taking into account the continuity equation

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 .
$$

Here $k$ and $\sigma$ are average filtration coefficient and porosity in the aquifer, $g$ is the standard gravity, $p=p(x,y,z)$ is the pressure.

We assume that at the initial time moment the fluid in the productive layer is at rest, i.e.

$$
u(0,x,y,z)=w(0,x,y,z)=0,
$$

(2)
System (1) is a consequence of the Navier-Stokes equations under assumption, that the quadratic terms are not taken into account in view of the smallness. We will assume that the aquifer $\Omega$ (Fig. 1) is located between two unpenetrable layers, bounded by the parallel planes $z=Z_{11}$ and $z=Z_{22}$. On these surfaces for pressure we set the boundary conditions

$$\frac{\partial p}{\partial z} \bigg|_{z=Z_j} = 0, \ j = 1, 2. \quad (3)$$

Similar conditions are given on the lateral surfaces of the computational domain $x=X_1, x=X_2, y=Y_1, y=Y_2$,

$$\frac{\partial p}{\partial x} \bigg|_{x=X_j} = \frac{\partial p}{\partial y} \bigg|_{y=Y_j} = 0, \ j = 1, 2. \quad (4)$$

Denote the cylindrical surface of the injection well with radius $r_1$ and the cylindrical surface of the production well with radius $r_2$ by $\gamma_1$ and $\gamma_2$, respectively ($Z_1 < z < Z_2$). On these surfaces, we set the pressures

$$P(t, x, y, z) \bigg|_{\gamma_1} = P_i - \rho g z, \quad P(t, x, y, z) \bigg|_{\gamma_2} = P_i - \rho g z. \quad (5)$$

In $\Omega$ excluding $\Omega_1$ and $\Omega_2$ the hydrostatic pressure, the pressure of the liquid column at a depth of $z$ is considered as an initial pressure $P(0,x,y,z)$

$$P(0, x, y, z) = -\rho g z. \quad (6)$$

To determine the pressure in $\Omega$ we consider the piezoconductivity equation

$$\frac{\partial p}{\partial t} = \omega \lambda \rho \Delta p, \quad (7)$$

For equation (7) the boundary and initial conditions (3)-(6) are given. Equation (7) is solved together with system (1) and initial condition (2). The pressure distribution in the aquifer $\Omega$ is obtained as a solution of the problem (1)-(7) and used to get the velocity field $(u, v, w)$.

Let $T=T(t,x,y,z)$ be a temperature distribution in $\Omega$ at the time moment $t$. The heat transfer in $\Omega$ will be carried out in two ways: convective and diffusion. Then the equation for the temperature in the aquifer will have the form

$$\frac{\partial T}{\partial t} + b \left( \frac{\partial T}{\partial x} u + \frac{\partial T}{\partial y} v + \frac{\partial T}{\partial z} w \right) = \lambda \Delta T, \quad (8)$$

which have be considered together with the equations for the velocity components $(u, v, w)$ of the fluid filtration in a porous soil,

and where $\lambda = \frac{\kappa}{\rho c \sigma (1-\sigma) + \rho c \sigma}$, $b = \frac{\sigma \rho c}{\rho c (1-\sigma) + \rho c \sigma}$, $\rho_0$ and $\rho_f$ are density of aquifer soil and of water, $c_0$ and $c_f$ are specific heats of aquifer soil and of water, $\kappa_0$ is thermal conductivity coefficient of soil, $\sigma$ is porosity. The aquifer has an initial temperature

$$T(0, x, y, z) = T_0(x, y, z). \quad (9)$$

In injection well temperature is set as “cold water” with temperature $T_i$, which returns from production well after using. At the surface $\gamma_1$ of $\Omega$, we will set the temperature of injected water

$$T(t, x, y, z) \bigg|_{\gamma_1} = T_i. \quad (10)$$

At the initial time on the surface $\gamma_2$ the temperature is equal to the temperature (9). Subsequently, during calculations, the temperature at the producing well $T_2(t)$ will vary due to the fact that the injected water has a lower temperature $T_i$. On the planes $z=Z_j$ we set a geothermal heat flux

$$\frac{\partial T}{\partial z} \bigg|_{z=Z_j} = \Gamma_j, \ j = 1, 2. \quad (11)$$

On the lateral boundaries we will use zero heat flux conditions.
Thus, for the temperature \( T(t,x,y,z) \) described by equation (8) all the necessary initial and boundary conditions (9)-(12) are given.

**Parallel algorithm**

The approach to construction of the parallel algorithm for multicore CPU using OpenMP technology was proposed in work (Akimova et. al., 2018).

The temperature and pressure computation procedure consist of three steps of forming and solving SLAEs for each spatial direction. Since each SLAE within one direction may be solved independently, we can distribute these SLAEs between several threads.

**Numerical experiment**

Numerical simulations are carried out for geothermal loop system, consisting of two wells. Figure 1b illustrates the water moving from injection to producing well.

The experiments were carried out using the six-core AMD Ryzen 5 1600X CPU. Numerical grid size is 237x157x21, time step is 24 hours and time interval is 1 year.

Table 1 shows the computing times \( T_p \), speedup \( S_p = T_p/T_1 \), and efficiency \( E_p = S_p/p \) for solving the problem by \( p \) threads. It also contains the theoretical speedup calculated using the Amdahl law

\[
S_p\text{th} = \frac{1}{\alpha + \frac{1-\alpha}{p}}
\]

where \( \alpha = 0.27 \) is the proportion of the serial code.

<table>
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<tr>
<th>Number of threads ( p )</th>
<th>Calculation time ( T_p ), seconds</th>
<th>Theoretical speedup ( S_p\text{th} )</th>
<th>Speedup ( S_p )</th>
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**Conclusions**

Thus, the developed mathematical model and the developed numerical algorithm allow to carry out simulations of temperature fields in an aquifer for various variants of open geothermal loops, consisting of two or three wells. The OpenMP approach allows one to get a speedup of 2.5 times using a six-core processor.

**Acknowledgements**

This work was partly supported by the Russian Foundation for Basic Research (project no. 19-07-00435).

**References**


Vaganova, N. A., Filimonov M. Yu. [2015] Simulation and numerical investigation of temperature fields in an open geothermal system. Lecture Notes in Computer Science, 9045, 393-399. https://doi.org/10.1007/978-3-319-20239-6_44


