Construction of the Earth's crust rheological boundaries based on a complex solution of inverse problems of magnetometry and gravimetry

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SUMMARY

According to the observed data, we had made a complex interpretation of the magnetic and gravitational fields of the Earth's crust region of the Middle and Southern Urals. The distribution of magnetization and density is calculated. To solve the structural magnetic inverse problem, a modified iterative local correction method is proposed. In this paper we describe new results of construction of the Earth's crust magnetic and density models (Pre-Urals, including the dilatant zone).
Introduction

When constructing a magnetic model, the geological concept is important. This article attempts to isolate blocks of the Earth’s crust that are heterogeneous in structure with different rheological properties (porosity, permeability). To solve this problem, a combination of magnetic and gravitational data is used. As a result of long-term studies of the geomagnetic field in the Middle and Southern Urals, a series of submeridional extended anomalies of the secular variations (ASV) have been identified. In our work, we consider the Bashkir ASV, which spatially coincides with the structures of the Pre-Ural marginal depression (Pyankov and Shapiro, 1986). A distinctive feature of the extended submeridional zone of the Bashkir ASV is the change in the sign of the increments $\delta (\Delta T)$ during the observation period. Anomalous changes $\delta (\Delta T)$, 10-15 nT/year, reach the highest intensity in the north of the study area (Krasnyi Zilim). These changes occur against the background of a normal field with amplitudes of $\pm 2$ nT/year. Analysis of a static anomalous magnetic field showed the absence of rock blocks with anomalously high magnetic properties in the Earth’s crust in the zones of the ASV. This made it possible to exclude from consideration the piezomagnetic hypothesis of the origin of anomalous variations. There remains only a hypothesis about the connection of anomalous secular variations of $T$ with intraterrrestrial currents of electrokinetic nature. In (Pyankov and Shapiro, 1977), it was shown that the appearance of electrokinetic nature on the Earth’s surface is possible only if horizontal geoelectric inhomogeneities and boundaries with a jump in the flow potential coefficient occur in the Earth’s crust (the presence of rock blocks with anomalously high porosity). Previously, the authors found that the selected zone ASV spatially coincides with the site of increased electrical conductivity of the Earth’s crust (Pyankov and Shapiro, 1986). The unresolved problem of the presence (or absence) in the Earth’s crust of blocks of anomalously low density (high porosity and permeability of rocks), i.e. blocks with contrasting rheological properties. The proposed work is aimed at solving this problem.

Method of separation of anomalies from sources in the Earth’s crust

To divide the long- and short-wave components of the amplitude spectrum of anomalies, geophysicists utilize numerical methods of field simulation at various altitudes (Martyshko and Prutkin, 2003). The anomalous magnetic field has an integral character and contains components from all the sources located in the upper lithosphere. In order to extract the anomaly from the sources in different layers of the Earth’s crust, a technique based on subsequent upward and downward magnetic data continuation was used. Firstly, the field is continued upwards to level $H$, due to which the effects of the local near surface sources (up to a depth $H$) are significantly suppressed. In order to get rid of the influence of the local sources located in the horizontal layer between the surface and depth $H$, the field, which was calculated to the height level above the initial surface, was then continued downwards to the depth $H$. Since the downward continuation procedure is an ill-posed problem, the regularization was applied. Finally, the field was continued upwards again to the initial surface. The resulting field can be treated as the field of the sources that are located below the $H$ level. Subtraction of this field from the observed one gives the field that is generated by the layer. By repeating this procedure in different heights and depths, we could separate the fields generated by the layers within the corresponding boundaries.

Simulation of the magnetic field $U(x, y, z)|_{z=0}$ measured on the Earth’s surface area $D = \{(x, y) \in \mathbb{R}^2 : a \leq x \leq b, \ c \leq y \leq d\}$ upwards to the level $z = -H$ is made by the Poisson’s formula

$$U(x, y, -H) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{-H}{(x-x')^2 + (y-y')^2 + H^2} U(x', y', 0)dx'dy'. \quad (1)$$

Simulation of the magnetic field to the initial level $U(x, y, -H)$ is implemented by the first type of Fredholm equation

$$Ku = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{2H}{(x-x')^2 + (y-y')^2 + 4H^2} \mathcal{U}(x', y', H)dx'dy' = U(x, y, -H). \quad (2)$$
Solving the integral equation of the first kind (2) is an ill-posed problem, which requires application of the regularization methods. The operator of equation (2) is positive definite and self-adjoint; therefore we can apply the scheme of Lavrentiev. After the grid discretization of equation (2) and approximation of the integral operator by the quadrature formulas, the problem is reduced to solving the system of linear algebraic equations (SLAEs) with the symmetric matrix $K$. In the regularized form, the system is

$$(K + \alpha I)u = \mathbf{U},$$

where $I$ - is the identity matrix and $\alpha$ is the regularization parameter. For solving the SLAEs the iteratively regularized simple iteration method was used

$$u^{k+1} = u^k - \frac{1}{\lambda_{\text{max}}} \left[ (K + \alpha I)u^k - \mathbf{U} \right],$$

where $\lambda_{\text{max}}$ is the maximal eigenvalue of matrix $K + \alpha I$ (symmetric case);

The stopping criterion in the iterative processes (4) is the fulfillment of the condition $\frac{\|Ku^k - \mathbf{U}\|}{\|\mathbf{U}\|} < \varepsilon$ with a sufficiently small $\varepsilon$.

Method to solve the magnetic inverse problem

Let $\{x, y, z\}$ are the rectangular Cartesian coordinates with axis Z pointing downwards and plane XOY coinciding with the surface of the observations. We consider the two-layer model of the Earth's crust. Surface separating the upper and lower layers at a sufficient distance from the center of area goes to the asymptote. The vertical component of the magnetic induction $Z$ at the point $(x, y)$ on the ground surface of the contact surface, separating the layers with vertical magnetization, $I_1$ and $I_2$ is calculated by the formula:

$$Z(x, y) = \Delta I \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( \frac{z(x, y)}{\left( (x - x')^2 + (y - y')^2 + z^2(x, y) \right)^{3/2}} + \frac{H}{\left( (x - x')^2 + (y - y')^2 + H^2 \right)^{3/2}} \right) dx' dy',$$

where $z=z(x, y)$ - the equation of the surface separating the upper and lower layers, the $\Delta I$ - magnetization jump at the boundary layers, $H$ - horizontal asymptote.

We solve the integral equation (5) - the full nonlinear 3D inverse problem - for a function $z(x, y)$ determining the geometry of an unknown contact surface. To solve the equation (5) and find the function $z(x, y)$ programs have been developed based on the modified method of local corrections (Martyshko et al. 2010). Method of local corrections were proposed for the approximate solution of nonlinear inverse problems of gravimetry (Prutkin, 1986) and is based on the assumption that a change in the value of the field at some point most affected by the change in the nearest to a given point of the surface $S$, which is the boundary between the two layers with different physical properties. A solution is calculated from data automatically by successive approximations, without a time-consuming trial-and-error process. A contrast value of physical properties and the position of a horizontal reference plane should be specified to obtain a unique solution. The scheme of an iterative method of finding the borders of magnetized layers defined by the equation $z=z(x, y)$ is as follows. At each step, an attempt is made to reduce the difference between the given and the approximate values of the field at a given point is only due to changes in the values of the required function at the same point. Discretization of equation (5) leads to the following system of nonlinear equations:

$$c \sum_j K_{ij}(z_j) = U_{ih},$$

where $c$ - a weighting factor of the cubature formula, $U_{ih} = \Delta Z(x_{ih}, y_{ih}, 0)$ - left-hand side of equation (5), $z_j = z(x_j, y_j)$, $K_{ij} = K(x_i, y_j, x_j, y_j, z_j)$ - the integrand. As a result, we received an iterative formula for finding $z^{n+1}_j$: $z^{n+1}_j = \frac{z^{n}_j}{1 + \alpha \left( \frac{z^{n}_j}{|U_{ij} - U_{ij}|} \right)}$, where $\alpha$ - regularization parameter, $\{z^{n}_j\}$ - the values of the unknown function $z(x,y)$, $n$ - the number of iteration.
Examples

An array of magnetometry data measured over an area measuring 450 km × 350 km was processed for the Western and Central parts of the Southern Urals (Fig. 1). Initial data - maps of scale 1: 200000.

Figure 1 Magnetic field in the Pre-Urals and Central Urals (recalculated to 20 km)

The central part of this area spatially coincides with the Bashkir ASV (from 140 to 240 km in latitude and from 100 to 350 km in longitude), which borders the folded Urals from the west. In the same territory, a submeridional anomaly of the electrical conductivity of the Earth’s crust was discovered. To study the nature of the anomaly according to the real data (after eliminating the influence of near-surface inhomogeneities) by the method of local corrections, the problem of determining the topography of the surface $S$ separating the layers with a magnetization jump of 1 A/m for the position of the asymptotic surface is $H = 20$ km. In fig. 2 shows a part of the surface relief $S$.

Figure 2 Relief of surface $S$ according to magnetic data in zone ASV

In figure 3 for the same area, the relief of the density surface $S_1$, which separates the layers with a density jump $\Delta\sigma = 0.1$ g/cm$^3$, is presented. The calculations were carried out by Akimova E.N. according to $\Delta g$ from a map of scale 1:200000. An analysis of the morphology of the isolines of the surfaces $S$ and $S_1$ implies a conclusion about their practical similarity. These are submeridional subsidence of the relief of surfaces $S$ and $S_1$ with an amplitude of about 3 km. The relief of each surface has its own specific feature. So, for example, the density model is characterized by the presence of a contrasting hemispherical shape in its northern extremity (Krasnyi Zilim). For the magnetic model, a similar immersion is also observed in the northern extremity of the anomalous
zone. If our preliminary assumption about the presence of an anomalous dilatant zone in the Earth’s crust is true, then the hemispherical immersion for the density model is an active dilatant inclusion, hydrodynamic processes in which can generate electrokinetic currents, which are the source of anomalous magnetic variations at the epicenter of the Bashkir ASV. The dilatant inclusion is an isometric block of rocks of high permeability, placed in a significantly lower permeability medium. Therefore, the surface under study is a rheological boundary of rocks with different densities and permeabilities. These rocks are located in that part of the Earth’s crust, which is characterized by certain hydrostatic conditions, i.e. pore fluid may flow.

**Figure 3** The relief of the surface $S_1$ according to gravity data in the zone ABX (according to the calculations of Akimova E.N.)

**Conclusions**

Thus, with the creation of original algorithms for solving the inverse problem of magnetometry and gravimetry and a modern technique for their implementation on a computer, it became possible to correctly interpret the observed field. As a result of a comprehensive interpretation of the magnetic and gravitational fields in the Earth’s crust in the zone of the supposed flow of electrokinetic currents, a section of the immersion of the magnetized surface at a depth of 20-25 km (magnetization jump of 1 A/m) is identified. The immersion amplitudes of the surface of magnetized rocks practically coincide with the iso-depths of the depression of the rheological surface with a density jump of 0.1 g/cm$^3$. This low-density zone (intense hemispherical immersion) is located at the latitude of Krasnyi Zilim, where anomalously high variations of the secular variations of the geomagnetic field are recorded. It is assumed that this is a site of tectonically active dilatant inclusion, the dynamics and rheological properties of which determine the spatio-temporal characteristics of variations in the magnetic field of an electrokinetic nature.

**References**


