The method of 3-D dataset statistical simulation with «cubic» correlation function on Rivne NPP example

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SUMMARY

The article is devoted to using methods of random fields in 3-D area statistical simulation (Monte Carlo methods) in environmental geophysical monitoring problems. A new method has been devised to simulate random field in 3-D area with «cubic» correlation function, based on spectral decomposition, for investigation of chalk layer density on Rivne NPP industrial area territory. It has been considered the problem of statistical simulation of «noise» for chalk layer density realizations as random fields in 3-D space. It has been constructed the statistical model for the gauss random fields in three-dimensional space, with spherical correlation function. It has been received of random fields in 3-D area realization with «cubic» correlation function by using those models, formulating the algorithm and building programs.
Introduction

The problems of the random field simulation in 3-D area arise in solving the actual environmental geophysical monitoring problems. The model example is chalk layer density data on Rivne NPP industrial area territory in this paper. The difference between the card input density values and the trend is in most cases a homogeneous isotropic random field (Vyzhva, Z.O., Demidov, V.K., Vyzhva, A.S. 2014, 2019). The random component in 3-D area is proposed to modeling on the basis of spectral decomposition (Vyzhva, Z.O. 2003, 2011) with «cubic» correlation function in this paper. The model and statistical simulation algorithm of random fields in 3-D area with «cubic» correlation function using the spectral representation was considered. It is known (Chilès, J.P. and Delfiner, P., 2012), that the «cubic» covariance model is used for differentiable variables as well as potential fields in geological modelling. The potential-field method was designed by (Chilès, J.P. et al. 2005) to build 3-D geological models from data available in geology and mineral exploration, namely the geological map, structural data, borehole data and interpretations of the geologist, because most 3-D geological known modelling tools were designed for the needs of the oil industry and are not suited to the variety of situations arisen in other application domains of geology.

The spectral representation of homogeneous isotropic random fields, model and statistical simulation algorithm for «cubic» correlation function

We consider a real-valued homogeneous isotropic random field \( \xi(r, \theta, \varphi) \) on the 3-D area \( r, \theta, \varphi \) – spherical coordinates. It is known (Yadrenko, M. 1983; Vyzhva, Z. 2003) that square-mean continuous real-valued isotropic random field \( \xi(r, \theta, \varphi) \), which is on 3-D Euclidean space \( \mathbb{R}^3 \), admits the spectral decomposition by spherical harmonics.

The correlation function of the homogeneous isotropic random field \( \xi(r, \theta, \varphi) \) on 3-D area \( B(\rho) \) depends on distance \( \rho \) between the vectors \( x, y \in \mathbb{R}^3 \) (\( x = (r, \theta_1, \varphi_1) \), \( y = (r, \theta_2, \varphi_2) \)): \[
\rho = r \sqrt{2(1 - \cos \psi)} = r \sin(\psi/2), \quad \text{where} \quad \cos \psi = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\varphi_1 - \varphi_2). \]

However, it is used the spectral decomposition of this random field by solution problems of statistical simulation of random field realizations on 3-D space, on this figurate real-valued random variables. Let adduce that decomposition.

**Theorem.** Let a mean square continuous realvalued homogeneous isotropic random field \( \xi(r, \theta, \varphi) \) is on 3-D space with zero mean. Than this random field admits (Vyzhva, Z. 2011, p.210) the following spectral decomposition:

\[
\xi(r, \theta, \varphi) = \sum_{m=0}^{\infty} \sum_{l=0}^{m} \widetilde{c}_{m,l} P_m^l (\cos \theta) \left[ \zeta_{m,1}^l (r) \cos l\varphi + \zeta_{m,2}^l (r) \sin l\varphi \right],
\]

where \( P_m^l (x) \) is associated Legendre functions of degree \( m \), \( \widetilde{c}_{m,l} \) – constants sequences are calculated by the formula:

\[
\widetilde{c}_{m,l} = \frac{1}{2} \sqrt{\frac{(m+1)}{2(m+l)}} (2m+1), \quad v_l = \begin{cases} 1, & l \neq 0, \\ 2, & l = 0; \end{cases}
\]

random processes sequences \( \{ \zeta_{m,k}^l (r) \} \), \( k = 1, 2; m = 0, 1, 2, \ldots; N; l = 1, \ldots, m; \)

\[
\zeta_{m,k}^l (r) = \int_0^\infty J_{m+1/2}(\lambda r) Z_{m,k}^l (d\lambda), \quad \text{such that satisfying the following conditions:}
\]

\[
1) \quad M \zeta_{m,k}^l (r) = 0; \quad 2) \quad M \zeta_{m,k}^l (r) \zeta_{m,k}^r (r) = \delta_{l}^{l'} \delta_{m}^{m'} \delta_{k}^{k'}, \quad b_m (r), \quad (3)
\]

where \( \delta_{m}^{m'} \) – Kronecker symbol, \( b_m (r) \) – the spectral coefficients and \( \{ Z_{m}^l(\cdot) \} \) is a sequence of orthogonal random measures on Borel subsets from the interval \( [0, +\infty) \), i.e. \( E Z_{m}^l(S_1) Z_{m}^l(S_2) = \delta_{l}^{l'} \delta_{m}^{m'} \Phi(S_1 \cap S_2), \) for any Borel subsets \( S_1 \) and \( S_2 \), where \( \Phi(\lambda) \) is the
bounded nondecreasing function so-called spectral function.

The spectral density is defined as \( f(\lambda) = d \Phi(\lambda)/d\lambda \) and it is obtained by correlation function as integral:

\[
f(\lambda) = \frac{2}{\pi} \int_0^\infty \rho \lambda \sin(\lambda \rho) B(\rho) d\rho
\]

The spectral coefficients \( b_m(r) \) are defined by the spectral density \( f(\lambda) \) of the isotropic random field \( \xi(r, \theta, \phi) \) on 3-D space in the way:

\[
b_m(r) = \int_0^\infty \frac{J_{m+\frac{1}{2}}(\lambda r)}{\lambda r} f(\lambda) d\lambda.
\]

The statistical simulations of random fields on the 3-D space realizations on the basis of spectral decomposition (1) are considered.

Approximation model is constructed by using the partial sums of series (1) and is the formula:

\[
\xi_n(r, \theta, \phi) = \sum_{m=0}^N \sum_{l=0}^m c_{ml} P^m_l(\cos \theta) \left[ \xi_{m,1}(r) \cos l\phi + \xi_{m,2}(r) \sin l\phi \right], N \in \mathbb{N}.
\]

We need the mean square approximation of random field \( \xi(r, \theta, \phi) \) by model (6) in the convenient form for the constructing statistical simulation of homogeneous isotropic random field realizations on the 3-D space algorithm.

Further we used the mean square estimate from (Vyzhva, Z. and Fedorenko, K. 2013) of the mean square approximation of random fields on the 3-D space.

Note, that we described the algorithm for the statistical simulation of realizations of Gaussian homogeneous isotropic random fields \( \xi(r, \theta, \phi) \) on the sphere \( S_3(r) \), which was constructed on the basis of model (6) and estimate from (Vyzhva, Z. and Fedorenko, K. 2013), but for Bessel type correlation function, on papers (Vyzhva, Z. et al. 2018).

We constructed on this paper the algorithm for the statistical simulation of Gaussian isotropic random fields \( \xi(r, \theta, \phi) \) on 3-D space with «cubic» correlation function:

\[
B(\rho) = \begin{cases} 
1 - 7 \left( \frac{\rho}{a} \right)^3 + \frac{35}{4} \left( \frac{\rho}{a} \right)^5 + \frac{7}{2} \left( \frac{\rho}{a} \right)^7 + \frac{3}{4} \left( \frac{\rho}{a} \right)^9, & \rho \leq a; \\
0, & \rho > a.
\end{cases}
\]

The spectral density is obtained by «cubic» correlation function (7) as:

\[
f(\lambda) = \frac{2}{\pi^4} \int_0^\infty \rho \lambda \sin(\lambda \rho) \left( 1 - 7 \left( \frac{\rho}{a} \right)^3 + \frac{35}{4} \left( \frac{\rho}{a} \right)^5 + \frac{7}{2} \left( \frac{\rho}{a} \right)^7 + \frac{3}{4} \left( \frac{\rho}{a} \right)^9 \right) d\rho
\]

The spectral coefficients, which correspond to the «cubic» correlation function (7) of random field \( \xi(r, \theta, \phi) \), are calculated by the formula:

\[
b_m(r) = \frac{2}{\pi} \int_0^\infty \frac{J_{m+\frac{1}{2}}(\lambda r)}{\lambda r} \left[ \rho \lambda \sin(\lambda \rho) \left( 1 - 7 \left( \frac{\rho}{a} \right)^3 + \frac{35}{4} \left( \frac{\rho}{a} \right)^5 + \frac{7}{2} \left( \frac{\rho}{a} \right)^7 + \frac{3}{4} \left( \frac{\rho}{a} \right)^9 \right) \right] d\rho d\lambda.
\]

**Algorithm**

1. Natural number \( N \) (border of summation) is chosen according to necessary accuracy \( \varepsilon > 0 \) of approximation the model (6) mentioned below:

\[
\frac{2^{M+2} r^{2M+2} (N+1)!}{(2N+3)!} \mu_{2N+2} \leq \varepsilon,
\]

where

\[
\mu_{2N+2} = \frac{2}{\pi} \int_0^\pi \int_0^\rho \rho l \sin(l \rho) \left( 1 - 7 \left( \frac{\rho}{a} \right)^3 + \frac{35}{4} \left( \frac{\rho}{a} \right)^5 + \frac{7}{2} \left( \frac{\rho}{a} \right)^7 + \frac{3}{4} \left( \frac{\rho}{a} \right)^9 \right) d\rho d\lambda, K = K(a) - const.
\]
2. Calculate the spectral coefficients $b_m(r), m=0, 1, 2, \ldots N$ for the «cubic» correlation function (7) as integral (9).

3. Simulate the sequences of independent Gaussian normal random variables:

\[
\{ \xi_{m,k}(r) \}, \quad k = 1, 2; \quad m = 0, 1, 2, \ldots N; \quad l = 1, \ldots, m;
\]

that satisfying the following conditions (3) with spectral coefficients (9).

4. Calculate the realization of the stochastic random field $\xi(r, \theta, \phi)$ by formula (6) in given point $(r, \theta, \phi), i = 1, 2, \ldots, I; j = 1, 2, \ldots, G; p = 1, 2, \ldots, P$ on the 3-D observations area by means of substituting in it values from the previous items 1, 2 and 3, numbers N and sequences of Gaussian random variables.

5. Check whether the realization of the random field $\xi(r, \theta, \phi)$ generated in step 4 fits the data by testing the corresponding statistical characteristics (distribution and correlation function $B(\rho)$).

The statistical simulation of realizations of the Gaussian isotropic random fields $\xi(r, \theta, \phi)$ with «cubic» correlation function can be done by means of this algorithm.

Statistical simulation methods of random fields dataset in 3-D area with «cubic» correlation function on Rivne NPP example

Complex geophysical research was conducted on Rivne NPP area. The greatest interests among these monitoring observations are radioisotope study of density and moisture of soil built along the perimeter of buildings. In this case was a problem addition by simulation of data that is received at the control density variance of chalky strata in the territory of industrial area investigated using radioisotope methods on a grid that included 29 wells. Simulation was performed at three levels (28, 29, 30 meters from the surface).

While constructing data graphs for each account, we noticed that it is expedient to distinguish deterministic and random components. Input data on the each level from the surface is a random field $\eta(x, y, z_i), i = 1, 2, 3; z_1 = 28m, z_2 = 29m, z_3 = 30m$. $\eta(x, y, z) = \eta_i(r, \theta, \phi)$ (r,0,\phi) – spherical coordinates, i – level number. The stationary random component $\xi_i(r, \theta, \phi)$ (frequently homogeneous isotropic random field) and trend $f_i(r, \theta, \phi)$ were selected for each level:

\[
\eta_i(r, \theta, \phi) = f_i(r, \theta, \phi) + \xi_i(r, \theta, \phi), i = 1, 2, 3.
\]

The final stage was the imposing array of realizations $\xi_i(r, \theta, \phi), i = 1, 2, 3$. what we got by statistical simulation in add points from observations area on the approximation of real data. As a result, we received more detailed implementation for the chalk layer density data in the selected area.

We received noise implementations on the study area with double detail, according to the algorithm. The built variogram (Fig. 1) of these implementations has the best approximation by theoretical variogram which is connected to the «cubic» correlation function (7) for parameter $a \approx 1, 25 \times 10^{-2}$ (a mean square deviation is $4, 35 \times 10^{-2}$). Note, that we described on paper (Vyzhva, Z. et al. 2014) the statistical simulation algorithm for this random field $\xi(r, \theta, \phi)$, which was constructed on the basis another model and with Bessel type correlation function.

This confirms the adequacy of simulated implementations to the real research data.

The spectral coefficients, which correspond to the «cubic» correlation function (7) of random field $\xi(r, \theta, \phi)$, are calculated by the formula (9) with parameter $a \approx 1, 25 \times 10^{-2}$.

These spectral coefficients we used in proposed above algorithm. The statistical simulation of realizations of the Gaussian isotropic random fields $\xi_i(r, \theta, \phi), i = 1, 2, 3$ with «cubic» correlation function (7) can be done by means of this algorithm.

The built variogram of implementations on the study area (Fig. 2) has adequate approximation by theoretical variogram which is connected to the «cubic» correlation function (a mean square deviation is $1, 36 \times 10^{-3}$).
Conclusions

The statistical simulation method of random field on 3-D area with «cubic» correlation function implementations makes it possible to supplement with a given detail the measurement results of chalk layer density data on Rivne NPP industrial area territory. It can also be used to detect abnormal areas. There are several other objects of statistical simulation methods application in geosciences. Among them primary are soil science and environmental magnetism (Menshov, O., et al. 2015).

References


