GeoTerrace Conference

EAGE

GeoTerrace-2020-007

Application of geopotential numbers to determine the heights of GNSS network reference stations

R. Pylypiuk, R. Pylypiuk, T. Hrytsiuk, *L. Dorosh, O. Hera *(Ivano-Frankivsk National Technical University of Oil and Gas)*

SUMMARY

The aim of the work is to study the possibility of using geopotential numbers to determine the heights of the reference states of the GNSS network. According to the results of regular GNSS observations at reference stations, their spatial coordinates X, Y, Z or B L, H for a certain epoch are determined. These coordinates related to the corresponding accepted reference ellipsoid and are thus spatial geocentric coordinates. Since the height of points in this system is a geodetic height, their use is not possible when mapping territories. Studies suggest the possibility of using geopotential numbers determined in the corresponding normal gravitational field to calculate elevations and normal heights. Based on the location of GNSS reference stations - observations of topographic maps (approximate spatial geodetic coordinates: latitudes and longitudes with an accuracy of 1', height - up to 1 m), geopotential numbers and their differences are calculated. The excess between the reference stations is calculated, their equilibration is performed and the values of the normal heights of these stations are calculated. The proposed technique allows to determine the normal heights of points on the earth's surface without geometric or trigonometric leveling, ie it is possible to determine the heights of points by a new gravimetric leveling method.

Introduction

The heights of the points are important characteristics of the spatial position of the physical surface of the Earth. Calculated from the level surface of the real potential W = W₀ (geoid) distance along the vertical line TM = to the point of the physical surface of the Earth is called orthometric height (Fig. 1), and calculated from the level surface of the normal potential U = W₀ (quasi-geoid) distance along a

vertical line TK′ = H^{γ} was called normal height. If the heights of the points of the physical surface of the earth are subtracted from the surface of the general terrestrial ellipsoid or reference ellipsoid, then we obtain the geodetic heights H and, accordingly, the geodetic system of heights.

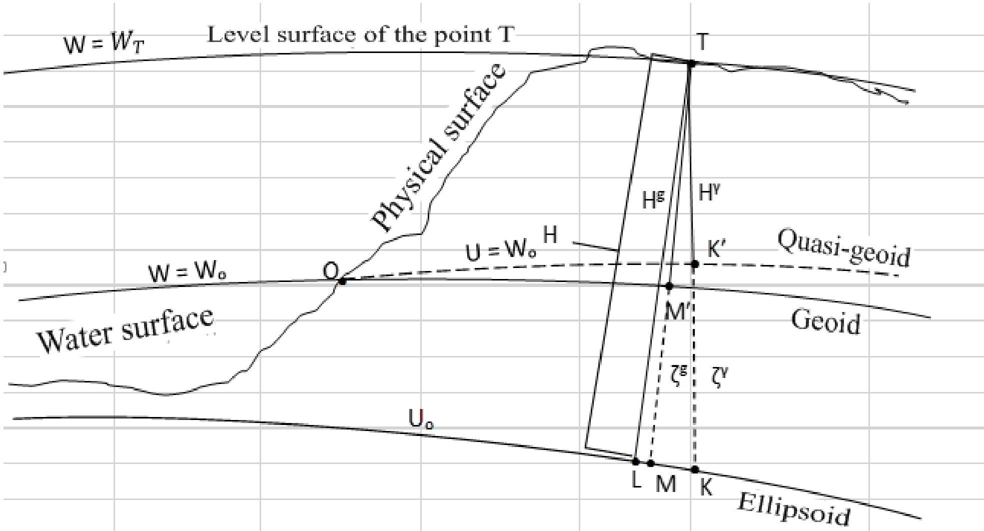

Figure 1. *Altitude systems in the Earth's gravitational field*

Calculation of orthometric and normal heights is carried out by formulas (Pellinen L.P, 1978):

$$H^g = \frac{1}{g_m}\int_o^T gdh \text{ and } H^{\gamma} = \frac{1}{\gamma_m}\int_o^T gdh, \tag{1}$$

where g_m та γ_m – respectively, the actual and normal average value of the force of gravity between the geoid (quasi-geoid) and a point on the physical surface of the Earth, and the component

$$\int_o^T gdh = C_T \tag{2}$$

called the geopotential or geopotential number of the corresponding point on the earth's surface.

Goal. The purpose of the study is to establish the possibility of using geopotential numbers to directly determine the elevations and normal heights of reference stations that form a geodetic network on the earth's surface, without traditional leveling.

Materials and Methods. Let the problem be to determine the excess between two arbitrary points i and j on the earth's surface. Consider the method of this definition in a normal system of heights based on formulas (1) and (2) (Kaula W. M., 2013). We write the expression for an arbitrary geopotential number C =. We form the difference of geopotential numbers for given points of terrain. We have:

$$\Delta C_{ij} = C_j - C_i = H_j^{\gamma}\gamma_{mj} - H_i^{\gamma}\gamma_{mi} \tag{3}$$

Replacing in this expression $H_i^{\gamma} = H_j^{\gamma} - h_{ij}$ after some transformations, we get:

$$h_{ij} = \frac{\Delta C_{ij}}{\gamma_{mi}} + \frac{\gamma_{mi} - \gamma_{mj}}{\gamma_{mi}} H_j^{\gamma}. \tag{4}$$

To implement the formula (4) we find its components, using the method proposed in the work "Geodetic systems in geodesy" (Marchenko OM et al., 2013). Determination of the average values of normal gravity at the study points and carried out according to a simplified formula used in Ukraine:

$$\gamma_m = \gamma_0 - \frac{0{,}3086}{2} \mathrm{H}^\gamma + \frac{0{,}072*10^{-6}}{2} (\mathrm{H}^\gamma)^2, \tag{5}$$

where γ_0 – normal value of gravity, calculated by the Helmert formula on the surface of the received ellipsoid by the value of the geodetic latitude of the observation point B. We have:

$$\gamma_0 = 978030(1 + 0{,}005302 sin^2 B - 0{,}000007 sin^2 2B). \tag{6}$$

Let's establish the necessary accuracy of definition of these sizes. Analyzing the formula (4), we establish that the component of the formula by which the non-parallelism of level surfaces is calculated is a small value, and therefore it can be ignored in the study of this issue.

$$m_{hij}{}^2 = 2\left(\frac{H_j}{\gamma_i}\right)^2 m_{\gamma m}^2 + 2\left(\frac{\gamma_j - \gamma_i}{\gamma_i}\right)^2 m_H^2. \tag{7}$$

In formula (7), the values of the root mean square error are set based on the differentiation of formula (5) taking into account (6). During the differentiation of formula (5) we assume that the component $\frac{0{,}072*10^{-6}}{2}(\mathrm{H}^\gamma)^2$ is a small value and can be neglected. We find:

$$m_{\gamma m}^2 = m_{\gamma 0}^2 + \left(\frac{k_2}{2}\right)^2 m_H^2, \tag{8}$$

where the vertical gradient of normal gravity k_2 = 0,3086 mGal / m. The value of the root mean square error of determining the normal force of gravity on the surface of the received ellipsoid is established on the basis of Helmert's formula. We have:

$$m_{\gamma 0}^2 = (k_0 k_1)^2 sin^2 2B \frac{m_B^{"2}}{\rho^{"2}}, \tag{9}$$

де $k_0 = 978030$ mGal, а $k_1 = 0{,}005302$.

Taking into account formulas (8) and (9), the root mean square error of determining the excess in this way can be established by the formula

$$m_{hij}{}^2 = 2\left(\frac{H_j}{\gamma_i}\right)^2 (k_0 k_1)^2 sin^2 2B \frac{m_B^{"2}}{\rho^{"2}} + \frac{2}{\gamma_i{}^2}\left[H_j^2 \left(\frac{k_2}{2}\right)^2 + (\gamma_j - \gamma_i)^2\right] m_H^2. \tag{10}$$

Based on formula (10), you can calculate the accuracy of the initial values of the geodetic latitude B and the normal height H to determine the excess with a given accuracy. We require the definition of excess with an accuracy of 1 cm. We will consider geodetic latitude as initial parameters *B*= 48°, normal value of gravity γ_i = 980980 mGal, normal height $H_j = 350$м, the difference between the normal values of gravity $\gamma_j - \gamma_i = 400$ mGal. Then, according to the principle of equal influence, we find that the error in determining the geodetic latitude can be $m_B = 9'$, and the error in determining the height $m_H = 12$м. These data indicate that at the initial stage of processing the GNSS network as a level, the initial values of geodetic latitudes and heights of network points can be determined by topographic maps of the appropriate scale.

Consider the possibility of applying the proposed method for determining normal heights. The original coordinates of these points, determined by the topographic map are presented in table 1.

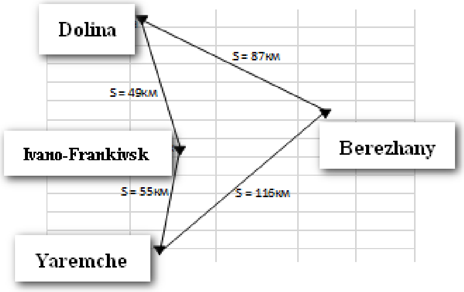

Figure 2. *A fragment of the satellite network of the company "System solutions"*

Table 1*. Initial geodetic data*

Name of items	Geodetic coordinates		Normal heights (m)
	Latitude *B*	Longitude *L*	
FRKV	48° 56'	24° 42'	274
DLHA	48° 58'	24° 02'	410
BRGH	49° 28'	24° 57'	316
YARM	48° 27'	24° 33'	548

In addition, the exact value of normal altitude is known for the Ivano-Frankivsk (FRKV) point H = 274,0897 m.

The processing algorithm consists of two stages: the first - is to determine the initial data on the basis of formulas (6) and (5) values of normal gravity, and by formula (3) geopotential numbers and their differences; the second – in the calculations calculated by formula (4), their equilibration and determination of normal heights. The results of the definitions in the first stage are shown in table 2.

Table 2*. Calculation of geopotential numbers and their differences*

Name of items	Parameters of the normal gravitational field			
	γ_{io}, mGal	γ_im, mGal	C_i, mGal*m	ΔC_i, mGal*m
FRKV	980970,9058	980928,6165	268862430,2	
DLHA	980973,8961	980910,6392	402173362,1	133310931,9
BRGH	981018,6931	980969,9379	309986500,4	-92186861,7
YARM	980927,4958	980843,1045	536521178,2	226534677,8
FRKV			268862430,2	-267658748

The results of calculations of elevations and heights in the second stage are shown in table 3.

Table 3*. Calculation of excesses and heights of points*

Name of items	Excess, m	Amendments, m	Fixed excess of m	Calculated heights, m	Accurate value height	Differences height
FRKV				274,0897	274,0897	0,0000
DLHA	135,9078	0,0099	135,9177	410,0074	409,9368	0,0706
BRGH	-94,0057	0,0175	-93,9882	316,0193	316,1411	-0,1218
YARM	230,9701	0,0234	230,9935	547,0128	547,5088	-0,4960
FRKV	-272,9341	0,0110	-272,9231	274,0897		
	-0,0618				m =	0,2578

fh_{valid} ± 0,0876

Since the processed geodetic network forms a closed structure, the sum of the calculated excesses determines the height invisibility $f_{h.}$ = - 0,0618м.

This shows that $f_{h\ valid} = \pm 5\sqrt{L}mm = \pm 0{,}0876\ m$ the accuracy of the stroke residual corresponds to the second leveling class.

By calculating the differences between the calculated and exact values of the heights of the corresponding reference stations, we obtain the value of the true errors, which makes it possible to assess the accuracy of their determination by the Gaussian formula (*m* = 0,258mm). Thus, in the example studied by the fourth approximation, improved results were obtained, which are shown in tables 4 and 5.

Table 4. *Calculation of geopotential numbers and their differences in the fourth approximation*

Name of items	Altitude points	Parameters of the normal gravitational field			
		γ_{io}, mGal	γ_im, mGal	C_i, mGal *m	ΔC_i, mGal *m
FRKV	274,0897	980970,9058	980928,6165	268862430,2	
DLHA	410,112	980973,8961	980910,6219	402283218	133420787,8
BRGH	316,1475	981018,6931	980969,9151	310131153,9	-92152064,2
YARM	547,128	980927,4958	980843,0847	536646747,8	226515594
FRKV	274,0897			268862430,2	-267784317,6

Table 5. *Calculation of excesses and heights of points in the fourth approximation*

Name of items	Excess, m	Amendments, m	Fixed excess, m	Calculated heights, m	Accurate value height	Differences height
FRKV				274,0897	274,0897	0,0000
DLHA	136,0198	0,0099	136,0297	410,1194	409,9368	0,1826
BRGH	-93,9702	0,0175	-93,9527	316,1667	316,1411	0,0256
YARM	230,9507	0,0234	230,9741	547,1408	547,5088	-0,3680
FRKV	-273,0621	0,0110	-273,0511	274,0897		
Нев'язка	-0,0618				m =	0,2058

fh_{valid} ± 0,0876

From the analysis of the data in table 5 we can conclude that, in general, the accuracy of determining heights with increasing number of iterations has improved (*m* = 0,206mm).

Conclusion

The possibility of using geopotential numbers to calculate elevations and heights of reference stations of the spatial geodetic network is considered.

It is established that according to the parameters of the normal gravitational field, the elevations are determined with the accuracy of geometric leveling of the second class, which allows to calculate the normal heights of reference stations, the distance between which is tens of kilometers.

The algorithm used to calculate elevations and heights allows the use of iteration techniques, which increases the accuracy of the final results.

The proposed technique allows to determine the normal heights of points on the earth's surface without geometric or trigonometric leveling.

References

Vanicek P. Vertical Datum and NAVD88 (1991). Surveying and Land Information Systems. Vol.51, number 2. p. 83-86. (in English).

Pellinen LP Higher geodesy (Theoretical geodesy). M., Nedra, 1978. 284p. (in Russian).

Marchenko O.M, Tretiak K.R., Yarema N.P. (2013) Reference systems in geodesy: textbook. manual. Lviv: Lviv Polytechnic Publishing House, 216 p. (in Ukrainian).

Kaula W. M. (2013). Theory of satellite geodesy: applications of satellites to geodesy. Courier Corporation. 142 p. (in English).