GeoTerrace-2020-057

# Peculiarities of the method of calculation feature's geodetic area on the reference ellipsoid in GIS

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## SUMMARY

The transition from the cartographic to the geoinformation approach to the production of topographic maps determines the transition from cartometric to analytical methods in GIS. Authors researched the peculiarities of calculation the geodetic area of a feature on the reference ellipsoid by Simpson's method. Peculiarities of calculating feature's geodetic area are determined by developing the special function in PostgreSQL 12.0. The programmed special function for calculating the geodetic areas of features depends on the number of segments involved in Simpson's method because it is affected by the specified accuracy of determining these areas.

## Introduction

The principle is observed that modern computer technologies provide the possibility of implementing high-precision rigorous mathematical methods during the researching of peculiarities of calculation the geodetic areas of features on the reference ellipsoid in the geoinformation environment. As mentioned earlier by the authors of the research (Karpinskyi & Kin, 2020), standard tools of geographic information systems (hereinafter – GIS) are not published in general or are not described enough for confirming the use of a method. It creates problematic issues when using such a GIS product, for example, for tasks in cadastral systems, determination of hydrological characteristics during the design of hydraulic structures, during the design of roads and road structures.

## Method and Theory

The transition from the cartographic to the geoinformation approach to the production of topographic maps determines the transition from cartometric to analytical operations in the geoinformation environment. In previous work (Karpinskyi & Kin, 2020), the authors of the research empirically determined that geodetic and cartometric operations in the implementation of standard GIS is usually hidden and it is like the "black box" in the process of determining the metric properties of geospatial features on the ellipsoid. It follows that the method of calculation cartometric characteristics are not open to the user, so it requires detailed researching.

The international experience in calculating the area on a reference ellipsoid in a geoinformation environment indicates the need to use a reference ellipsoid instead of cartographic projects (Karney, 2013; Karney, 2011; Chamberlain & Duquette, 2007; Obidenko, 2018; Kuźma & Pędzich, 2012; Baranovsky and all, 2009). The author of the article (Obidenko, 2018) noted that the peculiarity of determining the metric characteristics of features on the ellipsoid is that the exact formulas for calculating the area of polygonal features on the ellipsoid do not exist, explaining this issue due to low representation of tools for measurements in geodetic coordinate systems in modern GIS. Currently, the State Enterprise "Research Institute of Geodesy and Cartography" conducts research on cartometric operations on the ellipsoid in the environment of various GIS, in particular the determination of areas of the territory on the ellipsoid and others. In the research (Baranovsky and all, 2009), the obtained results concluded that it is necessary to develop software components for the analytic method of determination areas and assessment of their accuracy in GIS. It is also necessary to execute a research of the calculation areas in the existing land management documentation and to develop guidelines for the implementation of accurate methods for calculating the area in the land management documentation.

The goal of the research is to determine the peculiarities of the method of calculation feature's geodetic area on the reference ellipsoid in a geoinformation environment using the functions of its calculating in PostgreSQL. The geodetic area of a geospatial feature is the area of the geospatial feature, determined by the coordinates of the vertices of the boundaries on the reference ellipsoid.

## Results

There are methods of numerical integration in the core of all methods of determining the area. For some subintegral functions, the integral can be calculated analytically, but in the general case, the subintegral function may not be defined or the subintegral functions may not be elementary. It leads to the need to develop a function due to numerical methods that will determine any parameters with sufficient accuracy. The methods of numerical integration are used in the research: the contour integration along by segments and the approximate contour integration. The first method has been previously researched (Karpinskyi & Kin, 2020), so authors considered the second in more detail. This research is based on the contour integration by the method of summing of curvilinear trapezoids. This method was considered in more detail. It was supposed that some triangle ABC is given on the ellipsoid (Figure 1). The formula (1) is determined that the calculation the area of the features is reduced to the algebraic sum of the areas of curvilinear trapezoids.

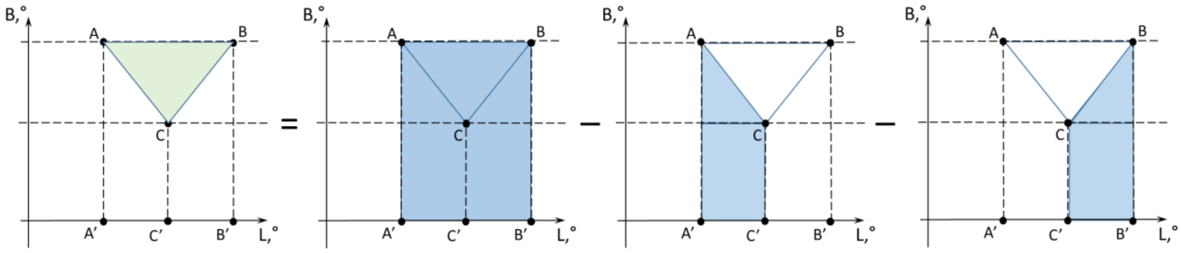


***Figure 1** Scheme for the calculation the area of a feature as an algebraic sum area of curvilinear trapezoids*

$$S = S_{ABB'A'} - S_{ACC'A'} - S_{CBB'C'}, \qquad (1)$$

where $S$ – an area of triangle ABC on the ellipsoid;
$S_{ABB'A'}$ – an area of a curvilinear trapezoid ABB'A' on the ellipsoid;
$S_{CBB'C'}$ – an area of a curvilinear trapezoid CBB'C' on the ellipsoid;
$S_{ACC'A'}$ – an area of a curvilinear trapezoid ACC'A' on the ellipsoid.

The formula (2) determines the area of an elementary trapezoid on an ellipsoid, what bounded by the zero meridian and some given one:

$$S = MN \cos B dBL, \qquad (2)$$

where $M = \frac{a(1-e^2)}{\sqrt{(1-e^2 \sin^2 B)^3}}$ – the radius of curvature of the meridian; $N = \frac{a}{\sqrt{(1-e^2 \sin^2 B)}}$ – the radius of curvature in the prime vertical; a – the semi-axe of the ellipsoid; e – the eccentricity; *B*, *L* – a latitude and a longitude of polygon’s vertices.

So the area of a curvilinear trapezoid determinate by a curvilinear integral by formula (3):

$$S = \int_{B_i}^{B_{i+1}} MN \cos B dBL \cdot \qquad (3)$$

Such the integral is defined as parametric by Simpson's formula (4):

$$S = \sum_{i=0}^{k-1} \frac{\Delta}{6}(f_i + 4 f_{cp} + f_{i=1}), \qquad (4)$$

where *k* – the number of segments what is divided from the start point to the endpoint of the interval and what depends on the specified accuracy of calculations area and the specified minimum interval; Δ – a minimum interval’s length, step; *f* – the value of the area for the segment bounded by coordinates $B_i = B_{П} + (B_K - B_{П})t$, $L_i = L_{П} + (L_K - L_{П})t$; *t* – a parameter.

Considering the peculiarities of the approximated contour integration, the requirements for the accuracy of determining the area of land parcels in the land management documentation, it was developed the SQL-function for calculation feature’s area on a reference ellipsoid by the Simpson method in the environment of the PostgreSQL 12.0.

Before programming this method, the block diagram was constructed that shows the logical sequential steps of the program operations, using PostGIS methods (Figure 2). The calculation is performed using the SELECT command, calling the developed function.

The method of calculating the geodetic area of the feature is implemented using an iterative process of calculation by the Simpson method. Iteration of the calculation of the areas of the segments is executed until the difference between the areas of curvilinear trapezoids is a given accuracy in determining feature's area. If $f(x)$ – a simple function, there is an area $J_n(x)$ of $n$ segments [$a$, $b$], then necessarily the number of segments $2n$ to double and to calculate the new area $J_{2n}(x)$. If $|J_{2n}(x) - J_n(x)| \le \varepsilon$, then the calculation is completed, where $\varepsilon$ – given the accuracy of determining the geodetic area. Otherwise, the number of segments is doubled again until the required accuracy is achieved.

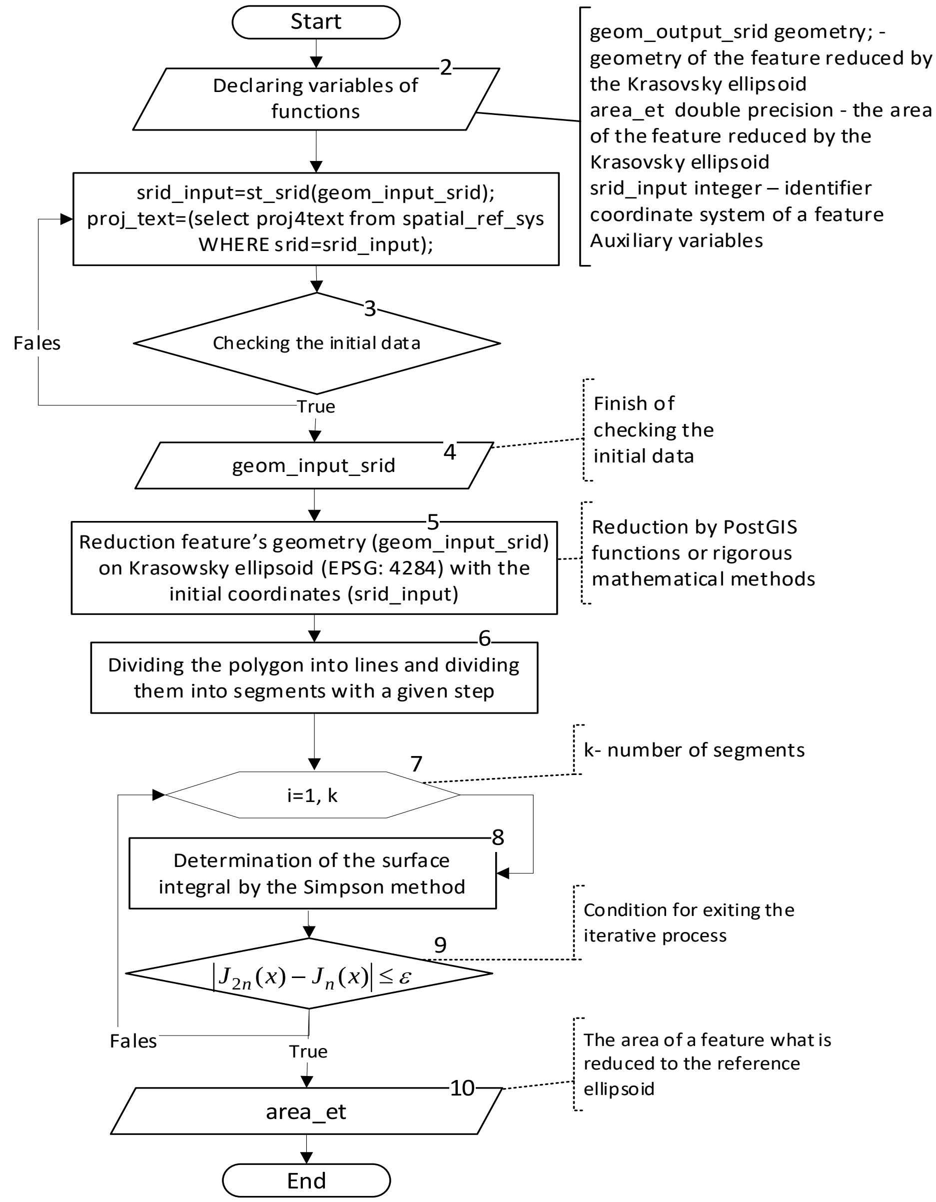


***Figure 2** The block diagram of the method of calculating the geodetic area on the ellipsoid by the method of the approximate contour integration*

According to the block diagram of the method after declaring variables and their types, the input data is checked. To calculate the area of the feature on the reference ellipsoid, the type of geometry: a polygon or a multipolygon, defines a constraint. Dates of all coordinate systems are stored by default in the table *public.spatial_ref_sys* after connecting the extension *postgis*. Coordinate system parameters must be in Proj4 format for convenient and automatic calling of dates and obtaining their values. The validated geometry of the feature must reduce by the Krasovsky ellipsoid depending on the input coordinate system. After the reduction, the data is prepared for the calculation by the

Simpson method, namely the division of the feature polygon on the lines of its boundaries and the specified number of segments *k*.

Next, the calculation of the area by the Simpson method of a given number of segments and return the result: the area of the geospatial feature on the surface of the Krasovsky ellipsoid.

The programmed special function for calculating the geodetic areas of features depends on the number of segments involved in the Simpson method because it is affected by the specified accuracy of determining these areas.

**Conclusions**

Peculiarities of calculation the geodetic area of a feature on a reference ellipsoid in a geoinformation environment are determined by developing the special function in PostgreSQL 12.0. The special function for determining the geodetic area of a feature after their reduction on the Krasovsky ellipsoid has been programmed in SQL.

In the following researches, based on the developed special function for calculating the geodetic areas of features by Simpson's method, the least-squares method will be considered, as well as determining the hydrological characteristics of hydrographic features (rivers, lakes, etc.) using analytical and numerical computer methods.

**Acknowledgements**

The implementation of the results of the work was performed in the State Enterprise "Research Institute of Geodesy and Cartography" following the certificate provided by them №379/1 on 14 December 2018.

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