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The Comparison of Random Fields with «Power» and Whittle-Matern Correlation Functions in 3D area by Statistical Simulation on Rivne NPP example

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SUMMARY

The article is devoted to using methods of random fields in 2D area statistical simulation (Monte Carlo methods) in environmental geophysical monitoring problems. A new method has been devised to simulate random field in 2D area with «power» correlation function, based on spectral decomposition, for investigation of chalk layer density on Rivne NPP industrial area territory. It has been considered the problem of statistical simulation of «noise» for chalk layer density realizations as random fields in 2D space. It has been constructed the statistical model for the gauss random fields with «power» correlation function in two-dimensional space.

It has been received of random fields in 2D area realization with «power» correlation function by using those models, formulating the algorithm and building programs. A comparative analysis of random fields with «power» correlation function with Whittle-Matern correlation function is given.

Key words: the statistical simulation, the random field, the «power» correlations function, the spectral decomposition, the statistical model.

Introduction

It is proposed to apply the method of random fields statistical simulation to solve the problems of conditional maps, adding of data to achieve the necessary precision, and other such problems in geophysics. It is solved to take the random component of data to modelling on the basis of the spectral decomposition of random fields on the plane using the built model and the involvement of the «power» correlation function in this paper. It is known (Chiles and Delfiner, 2012), that the «power» covariance model is used in geological modelling.

Model example - data on the density and moisture of the soil located on the perimeter of buildings on the territory of Rivne NPP. The soil density was determined by gamma-gamma logging. In this case was a problem additions data by simulation data that is received at the control density changes chalky strata in the territory of industrial area investigated using radioisotope methods on a grid that included 29 wells.

Theoretical aspects of capacity use of statistical simulation to solving problems in works of geophysics considered in (Yadrenko, 1983), (Grikh (Vyzhva) et al., 1993), (Vyzhva, 2003, 2011). Practical testing of statistical simulation on real data of density chalky strata on the territory of the Rivne NPP was carried out for the fields on the plane - in the (Vyzhva et al., 2004), but using only Bessel correlation function and Cauchy function (Vyzhva et al., 2017), and by using circular and Whittle-Matern correlation function (Vyzhva et al., 2019a, 2019b).

In this paper, the method, the model and procedure involving enough adequate data «power» type correlation function used. We considered the comparison between statistical simulation of random fields with «power» correlation functions and Whittle-Matern correlation functions in the geophysics problem of environment monitoring.

The model and statistical simulation algorithm for 2D random field with «power» correlation function

Data density chalky strata divided by deterministic and random components. Thus, the problem has been reduced to simulation of random component $\xi_N(r,\varphi)$, which is often homogeneous and isotropic random field. The using the authors' techniques of statistical simulation first implies preliminary statistical data processing to determine its statistical characteristics: the mathematical expectation and the correlation function.

The statistical model was chosen for the data correlation function of distribution of chalky strata density in the flat observation area. This function is defined by comparing the mean square approximation of the empirical and theoretical semivariogram. As result the most adequately input data was described by means of 5 types of correlation functions: the Bessel function, the Cauchy function, the Whittle-Matern function, the circular function and the «power» correlation function. The «power» correlation function is:

$$B(\rho)=\begin{cases}\left(1-\frac{\rho}{a}\right)^2, & 0\le\rho\le a,\\ 0 & ,\rho>a\end{cases}\quad a>0. \qquad (1)$$

The spectral density, which corresponding to the «power» correlation function (1), is:

$$f(\lambda)=\lambda\int_0^a \rho J_0(\lambda\rho)\left(1-\frac{\rho}{a}\right)^2 d\rho\,,\lambda\in R^1, a>0, \qquad (2)$$

where $J_\nu(\lambda\rho)$ - is the Bessel function of the first kind of order ν.

Figure 1 and Figure 2 presents plots of a semivariograms of the separated random data component of chalky strata density that according to the Whittle-Matern and the «power» correlation functions respectively.

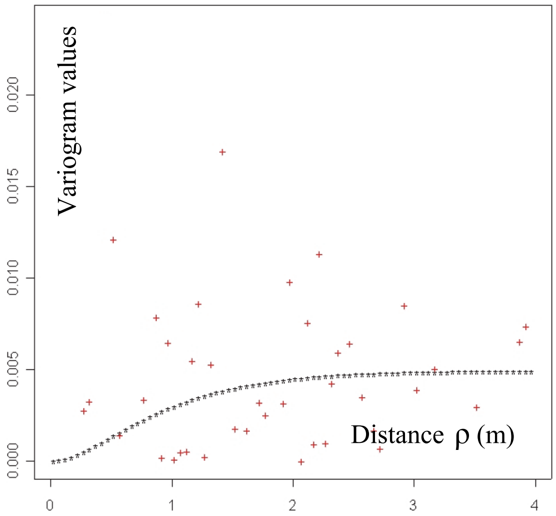


Figure 1 *Semivariogram of separated random component input data of the chalky strata density, that corresponding to the Whittle-Matern correlation function at* $\nu = 3/2$ *(mean square deviation - 0.000311)*

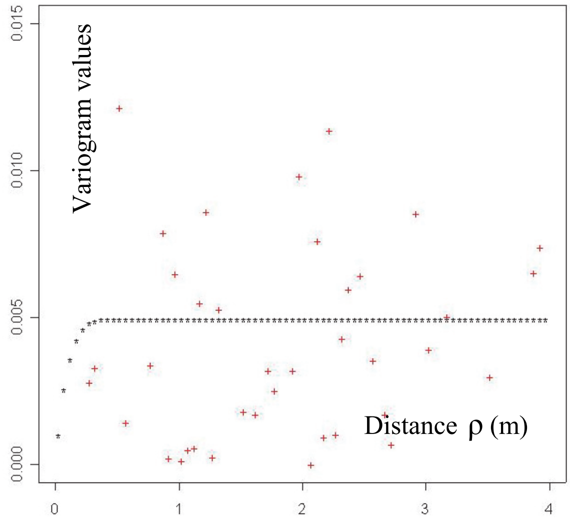


Figure 2 *Semivariogram of separated random component input data of the chalky strata density, that corresponding to the «power» correlation function at* $a = 9/2$ *(mean square deviation - 0.000232)*

The spectral coefficients, which according to «power» correlation function, are determined by calculating the integral:

$$b_k(r) = \frac{2}{\pi}\int_0^{\pi}\left(1 - \frac{2r\,sink\varphi}{a}\right)^2 cos2k\varphi\, d\varphi. \qquad (3)$$

The realizations of 2D random field with «power» correlation function (1) at the values of parameters $a = 9/2$ are generated. The statistical simulation was performed by using the technique of spectral decomposition and finding spectral coefficients.

From the spectral theory (Yadrenko, 1983) follows that the model of random fields on a plane with such correlation function in a sum of:

$$\xi_N(r,\varphi) = \sum_{k=0}^{N}\sqrt{\nu_k b_k(r)}\ \ [\zeta_k(r)cosk\varphi + \eta_k(r)sink\varphi]\,, \qquad (4)$$

where $\nu_k = \begin{cases}1, & k = 0\\ 2, & k > 0\end{cases}$, $b_k(r)$, $(k = 0,1,2,\dots,N)$ are the spectral coefficients in form of (3), which corresponding to the «power» correlation function (1);
r and φ $(r \in R_+,\ \varphi \in [0,2\pi])$ are polar coordinates of the point x on the plane (includes observation area), and the distance ρ between the points $x_1 = (r_1,\varphi_1)$ and $x_2 = (r_2,\varphi_2)$ is equals
$\rho = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\varphi_1 - \varphi_2)}$;
N is an integer number (the number of the terms in the model), the value of N is determined by the prescribed small number ε (approximation accuracy) by using the inequality from paper (Grikh (Vyzhva) et al., 1993), which is the estimate of the mean square approximation of random field $\xi_N(r,\varphi)$ by partial sums $\xi_N(r,\varphi)$.

The procedure of numerical simulation the realizations of the random component data field, by means of the abovementioned model (5), was conducted by using the Spectr 2.1 software, which is described in (Vyzhva et al., 2004).

The value of number N for the constructed model is determined by using the inequality, which is the estimate of the mean square approximation of random field $\xi(r,\varphi)$ by partial sums $\xi_N(r,\varphi)$. This number N corresponds to the prescribed small number ε (approximation accuracy). The mentioned inequality was obtained:

$$\mathrm{M}[\xi(r,\varphi)-\xi_N(r,\varphi)]^2 \le \frac{1}{\pi N}(r\mu_1 + r^2\mu_2), \tag{5}$$

where $\mu_k = \int_0^\infty \lambda^{k+1} \int_0^a \rho J_0(\lambda\rho)\left(1-\frac{2r\ \sin k\varphi}{a}\right)^2 d\rho d\lambda\,, k = 1{,}2.$

The statistical simulation procedure of Gaussian homogeneous isotropic random field $\xi(r,\varphi)$ on the plane with «power» correlation function was built by means of the model (4) and the estimate (5). This random field is determined by its statistical characteristics: the mathematical expectation and the «power» correlation function $B(\rho)$ (1) at the value of parameter $a = 9/2$.

Procedure

1) The positive integer number N is determined corresponding to the prescribed accuracy ε and by using inequality (5), where r is a polar radius of the point on the plane in which the realization of the random field $\xi(r,\varphi)$ is generated.

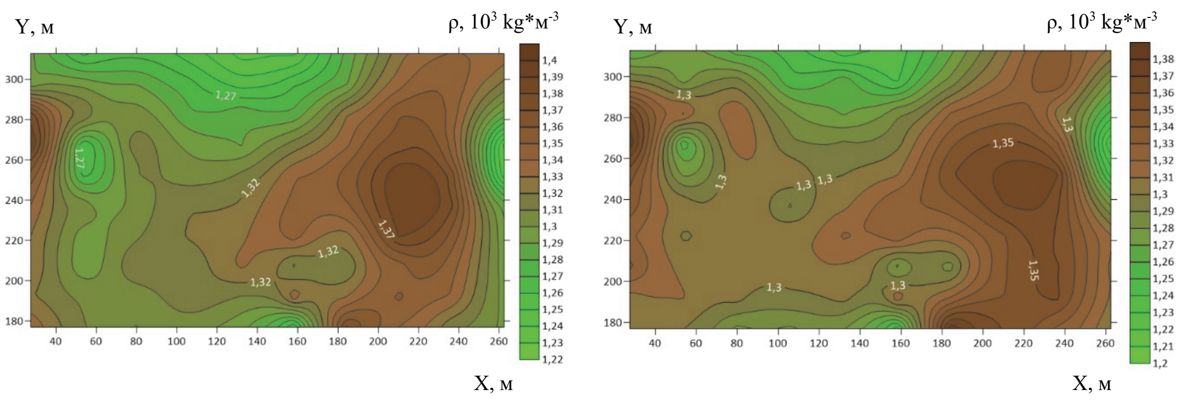
2) We calculate the spectral coefficients (3) of «power» correlation function at the value of parameter $a = 9/2$.

3) We generate values of the standard normal random variables $\{\zeta_k,\ k = 0,1,2,\dots,80\}$ and $\{\eta_k,\ k = 0,1,2,\dots,80\}$.

4) We evaluate the model expression (4) in the grid points by substituting in it values which were found in the previous steps.

5) The statistical estimate of the correlation function is obtained by the realizations of the random $\xi(r,\varphi)$. This estimate compares with a given «power» correlation function at $a = 9/2$ and provides the statistical analysis the adequacy of realization.

The simulation results (Figure 3) presents that the chosen model of the data is enough adequate. The developed Spectr 2.1 software (by Python) works with sufficient accuracy (mean square deviation - 0.0000168).



a b

Figure 3 *The maps of chalky strata density distribution on the industrial area of Rivne nuclear power plant at a depth of 28 m. from the surface, according to simulated data of 29 observational boreholes* *over 1984-2004 years that based on the values in secure boreholes by model with spectral coefficients of the Whittle-Matern type (a) and spectral coefficients of the «power» function (b)*

Conclusions

The theory, techniques and procedure of statistical simulation of 2D random fields with «power» correlation functions can significantly increase the effectiveness of monitoring observations on the territory of potentially dangerous objects. This makes it possible to simulate the values in the area between regime observation grids and abroad, adequately describe real geological processes.
The method of statistical simulation of random fields with «power» correlation functions allows complementing data with a given accuracy. We considered the comparison between statistical simulation of random fields with «power» correlation functions and Whittle-Matern correlation functions at the control density changes chalky strata in the territory of industrial area investigated. There are many other objects in geosciences for those statistical simulation methods application. Among them primary are soil science and environmental magnetism (Menshov et al., 2018).

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