High-accuracy integration algorithm for attitude heading reference system

*O. Sapehin, O. Pavlovskyi (Igor Sikorsky Kyiv Polytechnic Institute)

SUMMARY

Simulation model of modern microelectromechanical attitude heading reference system was created. It was based on using high-accurate two-step integration method for Bortz vector equation. It allows to increase accuracy of quaternion attitude algorithm. To compensate bias instability of microelectromechanical gyroscopes, Kalman filter was applied. It based on estimation of quaternion a gyroscope errors by adding accelerometers and magnetometers signals. Presented algorithm can be widely used for any type of controlled vehicle and UAVs.

Keywords: attitude heading reference system, inertial, navigation, Kalman filter, gyroscope
Introduction

Strapdown attitude heading reference systems (AHRS) are the basis of modern navigation systems for any type of vehicles. They provide complete and accurate information about the orientation (angular position) of the object. It is important to note that AHRS are completely autonomous and do not require external information (Savage, 2013). Due to their ability to accurately determine the angular position of the object in any range of angles and with a high frequency of information output, AHRS today are unsurpassed and have no alternative.

The appearance of micromechanical sensors on the market has caused significant interest in small-sized and relatively inexpensive orientation systems that have sufficient accuracy to solve new problems in various fields of application. These systems are used in aviation, space navigation, shipping (surface and underwater), as well as in special equipment, such as small unmanned aerial vehicles, robotics, stabilization and control systems (Avrutov et al., 2020).

However, the problem of miniature orientation systems is their insufficient accuracy. There are two approaches to solving this problem: compensating with systems built on other physical principles, or developing algorithms to improve accuracy with specific applications in mind. This work considers an algorithmic approach to increasing accuracy, which involves optimal estimation of errors and their subsequent compensation.

Altitude algorithm

Let’s connect body frame \( x, y, z \) to navigation frame \( \xi, \eta, \zeta \) as it shown on Fig.1.

The \( \zeta \) axis is pointed to vertical, \( \eta \) axis is pointed to north, \( \phi \) is a heading angle, \( \theta \) pitch, \( \gamma \) roll. Attitude heading reference system in mounted to the base and it sensitive axes coincide to body frame \( x, y, z \). There are many types of AHRS sensors, but the most commonly used are gyroscopes, accelerometers and magnetometers. They create the inertial measurement unit (IMU). For angular attitude determination the quaternion equation (1) is integrated.

\[
\begin{bmatrix}
\dot{\lambda}_0 \\
\dot{\lambda}_1 \\
\dot{\lambda}_2 \\
\dot{\lambda}_3
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
-\lambda_1 & -\lambda_2 & -\lambda_3 \\
\lambda_0 & -\lambda_2 & \lambda_1 \\
\lambda_2 & \lambda_0 & -\lambda_1 \\
\lambda_3 & -\lambda_1 & \lambda_0
\end{bmatrix} \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} + \lambda_0 \begin{bmatrix}
0 & -\omega_z & -\omega_y & -\omega_x \\
\omega_z & 0 & \omega_x & -\omega_y \\
\omega_y & -\omega_x & 0 & \omega_z \\
\omega_x & \omega_y & -\omega_z & 0
\end{bmatrix} \begin{bmatrix}
\lambda_0 \\
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}.
\]

(1)

where \( \lambda_0, \lambda_1, \lambda_2, \lambda_3 \) – attitude quaternion components, \( \omega_x, \omega_y, \omega_z \) – angular rate from IMU gyros.

Knowing the quaternion components it’s easy to calculate current angle, using (2).

\[
\psi = \arctg\left(\frac{2(-\lambda_0 \lambda_2 + \lambda_1 \lambda_3)}{\lambda_0^2 - \lambda_1^2 + \lambda_2^2 - \lambda_3^2}\right), \quad \gamma = -\arctg\left(\frac{2(\lambda_1 \lambda_3 - \lambda_0 \lambda_2)}{\lambda_0^2 - \lambda_1^2 - \lambda_2^2 + \lambda_3^2}\right), \quad \theta = \arcsin(2(\lambda_0 \lambda_1 + \lambda_2 \lambda_3)).
\]

(2)

Kalman filter developing

Microelectromechanical gyro sensors which a commonly used in low-price IMU have bad accuracy and huge bias instability. This provides significant errors to all AHRS work. One way of accuracy incising is the

\[ X = \begin{bmatrix} \lambda_e \\ \Delta B \end{bmatrix} \tag{3} \]

where \( \lambda_e \) – attitude quaternion error, \( \Delta B \) – gyroscope bias instability. State vector calculates by integration of state equation

\[ \dot{X}(t) = F(t)X(t) + W(t), \tag{4} \]

where \( F(t) \) – state matrix, \( W(t) \) – noise vector.

State matrix from Eq.4 form like so

\[ F(t) = \begin{bmatrix} -[\tilde{\omega} \times] & -\frac{1}{2}I_{3 \times 3} \\ 0_{3 \times 3} & \tilde{\omega} \end{bmatrix} \tag{5} \]

where \( 0_{3 \times 3} \) – zeros \( 3 \times 3 \) matrix, \( I_{3 \times 3} \) – ones \( 3 \times 3 \) matrix, \([\tilde{\omega} \times] = \begin{bmatrix} 0 & -\tilde{\omega}_z & \tilde{\omega}_y \\ -\tilde{\omega}_z & 0 & -\tilde{\omega}_x \\ \tilde{\omega}_y & \tilde{\omega}_x & 0 \end{bmatrix}\) – angular rate estimations.

Second stage extrapolation results are being clarified by measurement next equation

\[ Y = D \cdot X + V; \tag{6} \]

where \( V \) – noise vector of accelerometers and magnetometers measurement;

\[ D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2tgI & -2 & 0 & 0 & 0 \end{bmatrix} \]

\( I \) – magnetic latitude.

This procedure provides real-time step-by-step tracking of the state of an object using current measurements and information about the previous state and its uncertainty. To implement this procedure, we move from a continuous error model described by a differential Eq. 4 of state to an equivalent discrete model described by a difference equation of the form:

\[ X(k) = \Phi(k, k - 1)X(k - 1) + W(k - 1), \tag{7} \]

\( \Phi(k, k - 1) \) matrix can be approximately found by

\[ \Phi = I_{6 \times 6} + F(t) \cdot \Delta t, \]

where \( \Delta t \) – is a sensors rate step.

Covariation matrix \( P \) calculates on prognose state:

\[ P^- = \Phi \cdot P^+ \cdot \Phi^T + Q_d, \]

where \( Q_d \) is a covariation matrix of gyro noise dispersion.
Kalman gain calculates like so
\[
K = P^{-} \cdot D^{T} \cdot \left( D \cdot P^{-} \cdot D^{T} + R_{d} \right)^{-1}
\]  

where \( R_{d} \) – is a covariation matrix of accelerometers and magnetometers noise dispersion.

Correction of \( P^{-} \) matrix comes on next algorithm step
\[
P^{+} = (I_{6 \times 6} - K \cdot D) \cdot P^{-}.
\]

Result of estimation vector calculation becomes
\[
\hat{X}^{+} = \begin{bmatrix} \hat{\lambda}_{e}^{+} \\ \Delta \hat{B}^{+} \end{bmatrix} = K \cdot Y.
\]  

(9)

Attitude quaternion estimation on current step is a product previous quaternion to error quaternion of current step
\[
\hat{A}^{+} = \hat{A}^{-} \circ [1, \hat{\lambda}_{e}^{+}].
\]  

(10)

At the very end of the cycle, gyroscope drifts are also evaluated and compensated as:
\[
\hat{B}^{+} = \hat{B}^{-} + \Delta \hat{B}^{+}
\]
\[
\hat{\omega}^{+} = \hat{\omega}^{-} - \hat{B}^{+}.
\]

**Attitude algorithm**

For depending the attitude quaternion for the first step it’s need to use initial condition algorithms like analytic gyrocompassing procedure from (Meleshko et al., 2011), or by using external data. The main tusk is to make integration algorithm for Eq.1, which allows to find \( \hat{A}^{-} \) for attitude quaternion from Eq.10. Precision AHRS use next approach: quaternion increment calculates by rotation vector increment which much easy and accurate. Rotation vector, or Euler’s vector finds by integration of Bort’s equation (Savage, 1998)
\[
\frac{d\phi}{dt} = \omega_{p}^{ps} + \frac{1}{2} (\phi \times)\omega_{p}^{ps};
\]  

(11)

where \( \phi = [\phi_{x} \quad \phi_{y} \quad \phi_{z}]^{T} \) – Euler’s vector components, \( \omega_{p}^{ps} \) – angular rate vector.

Integration of Eq.11 gives us current \( \phi \) and can find current quaternion by:
\[
\lambda_{0} = c_{\phi_{2}}; \quad \lambda_{1} = \frac{\varphi_{x}}{\varphi} \cdot s_{\phi_{2}}; \quad \lambda_{2} = \frac{\varphi_{y}}{\varphi} \cdot s_{\phi_{2}}; \quad \lambda_{3} = \frac{\varphi_{z}}{\varphi} \cdot s_{\phi_{2}}.
\]  

(12)

where \( c_{\phi_{2}} = \cos \frac{\varphi}{2}, \ s_{\phi_{2}} = \sin \frac{\varphi}{2}, \ \varphi = \sqrt{\varphi_{x}^{2} + \varphi_{y}^{2} + \varphi_{z}^{2}}. \)

Work (Lazarev et al., 2018), consist of varied numbers of numerical integration methods for Eq.11. They all differ by accuracy order and step-size – the number of sensors read interrupts in one integration step.

Two-step algorithms were applied to current AHRS algorithms
\[
\phi(2h) = h(\omega_{0} + 4\omega_{1} + \omega_{2})/3 + h^{2}[4\omega_{0} \times \omega_{1} + \omega_{0} \times \omega_{2} + 4\omega_{1} \times \omega_{2}]/15;
\]

where \( \omega_{i} \) – angular rate on significant interrupt, \( h \) – integration step.

**Simulation**

For research developed AHRS algorithm were created simulation model in Matlab. It was used real signal from AHRS-10P made by Inertial Labs. The main characteristics presented in Table 1.
Table 1 AHRS-10P characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AHRS-10P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic accuracy of heading, amplitude</td>
<td>0.6 deg</td>
</tr>
<tr>
<td>Dynamic accuracy of pitch and roll, amplitude</td>
<td>0.08 deg</td>
</tr>
<tr>
<td>Bias instability gyroscopes, amplitude</td>
<td>35 deg/h</td>
</tr>
<tr>
<td>Bias instability accelerometers, amplitude</td>
<td>0.5 mg</td>
</tr>
<tr>
<td>Dimensions</td>
<td>90<em>27</em>26 mm</td>
</tr>
<tr>
<td>Mass</td>
<td>84 g</td>
</tr>
<tr>
<td>Interface</td>
<td>RS-232, RS-422, CAN</td>
</tr>
</tbody>
</table>

To make possible the accuracy analyse developed AHRS it was used precise stand information about system angles. It was compared to results of algorithm work (Fig.2).

![Chart](chart.png)

**Figure 3** Attitude angles: a) algorithm AHRS, b) stand

Conclusions

Algorithm of an attitude heading reference system were developed. In based on numerical integration of Bortz equation as increment of attitude quaternion. Correction of huge gyroscope bias instability was provided by adding optimal Kalman filter which uses accelerometer and magnetometer measurements. Simulation of AHRS was provide by using signals from middle-class accuracy AHRS-10P. Results shows adequate and accurate work of developed AHRS algorithm.

References


