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Relativistic Geodesy: How Atomic Clocks Measure Height

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SUMMARY

Relativistic geodesy is an emerging field at the intersection of general relativity and quantum metrology. Its physical basis is the gravitational redshift: a clock at a lower gravitational potential runs slower than an identical clock at a higher potential, with the fractional frequency shift given by $\Delta\nu/\nu_0 = \Delta U/c^2$. While this effect was long confined to fundamental tests of general relativity, the emergence of optical lattice clocks with fractional frequency stability approaching 10^{-18} has turned it into a practical geodetic tool known as *chronometric levelling*. This paper reviews the physical basis, historical development and practical implementation of chronometric levelling, and compares it with classical height-determination methods such as geometric levelling and the GNSS/geoid approach. Unlike classical levelling, whose accuracy degrades with the square root of the traverse length, chronometric levelling is essentially insensitive to the distance between stations. Demonstrated examples, including the comparison of transportable optical clocks at the Tokyo Skytree, already achieve centimetre-level height accuracy. We argue that continued development of transportable and satellite-borne optical clocks, together with ongoing efforts to refine national height systems and regional geoid models, will make chronometric levelling an increasingly important complement to classical geodetic techniques.

Keywords: Relativistic geodesy; Chronometric levelling; Optical atomic clocks; Gravitational redshift; Gravitational potential.

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Introduction

Classical geodesy determines heights and the gravitational potential using geometric levelling, gravimetry and satellite techniques (GNSS combined with a geoid model). These approaches have systemic limitations: error accumulation over long distances, inconsistencies between national height systems, and dependence on a dense network of ground infrastructure. The emergence of optical atomic clocks with fractional frequency instability at the 10^{-18} level has opened a fundamentally new, purely relativistic way of measuring differences in gravitational potential — chronometric levelling — which now forms a separate research direction known as relativistic geodesy.

The underlying physical effect is the gravitational redshift predicted by general relativity: a clock located at a lower gravitational potential runs slower than an identical clock located higher up (Bjerhammar, 1985). As long as clock accuracy was limited to the 10^{-14}–10^{-15} level (caesium and rubidium standards), this effect remained a subject for fundamental tests of general relativity rather than a practical geodetic tool. The development of optical lattice clocks based on strontium and ytterbium atoms over the past fifteen years has changed this fundamentally: comparing the frequencies of two such clocks today makes it possible to "feel" a height difference of about one centimetre (Mehlstäubler et al., 2018).

The concept of using precise clocks to determine the gravitational potential was first formulated by the Swedish geodesist Arne Bjerhammar in works published in 1975 and 1985, who proposed the following definition: the relativistic geoid is the surface on which identical clocks run at the same rate, and which lies closest to mean sea level (Bjerhammar, 1985). The idea was independently developed by the Finnish geodesist Martin Vermeer, who introduced the term "chronometric levelling" itself (Vermeer, 1983). For nearly three decades the concept remained a theoretical construct, since available clocks lacked sufficient stability. The turning point came in 2010, when a NIST group experimentally detected the gravitational redshift for a height difference of only a few tens of centimetres (Chou et al., 2010), confirming the practical feasibility of the method. The subsequent development of stabilized optical-fibre frequency-transfer links, capable of preserving comparison accuracy over continental distances without signal degradation, finally turned chronometric levelling from a laboratory demonstration into a field geodetic method (Mehlstäubler et al., 2018).

Method and Theory

In the weak gravitational field of the Earth, the fractional frequency shift between two clocks located at points with different gravitational potential is described by relation (1):

$$\Delta \nu / \nu_0 = \Delta U / c^2 \qquad (1)$$

where $\Delta\nu$ is the difference between the clocks' proper frequencies, ν_0 is the nominal frequency, ΔU is the difference in gravitational potential between the observation points, and c is the speed of light in vacuum (Mehlstäubler et al., 2018). Since near the Earth's surface $\Delta U \approx g \cdot \Delta h$ (g being the acceleration of gravity and Δh the height difference), equation (1) can be rewritten as (2):

$$\Delta \nu / \nu_0 \approx (g/c^2) \cdot \Delta h \qquad (2)$$

Substituting numerical values ($g \approx 9.81$ m/s^2, $c \approx 3\times10^8$ m/s) gives a fractional frequency shift of about 1.1×10^{-16} per metre of height difference (Takamoto et al., 2020). This means that confidently resolving a 1 cm height difference requires a frequency-comparison instability no worse than 10^{-18} — a level that current optical clocks already reach or closely approach (Mehlstäubler et al., 2018).

Practical implementation of the method involves placing (stationary or transportable) optical clocks at two or more observation sites, connected by a frequency-transfer channel — a stabilized optical-fibre link or, in the future, a satellite optical link (Fig. 2). The frequency comparison, accumulated over a suitable averaging interval, directly yields the gravitational potential difference ΔU between the sites, which is then converted into a difference of normal heights Δh.

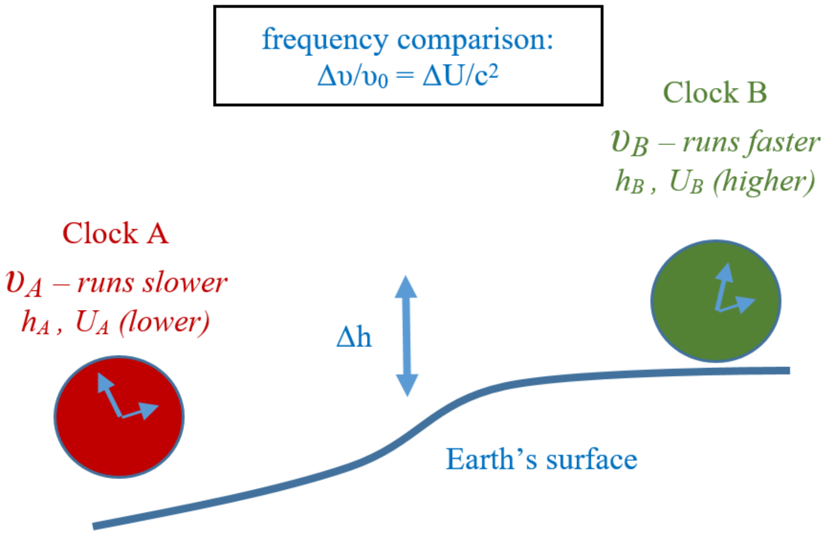

Figure 1. *Gravitational redshift: clock A (lower potential) runs slower than clock B (higher potential)*

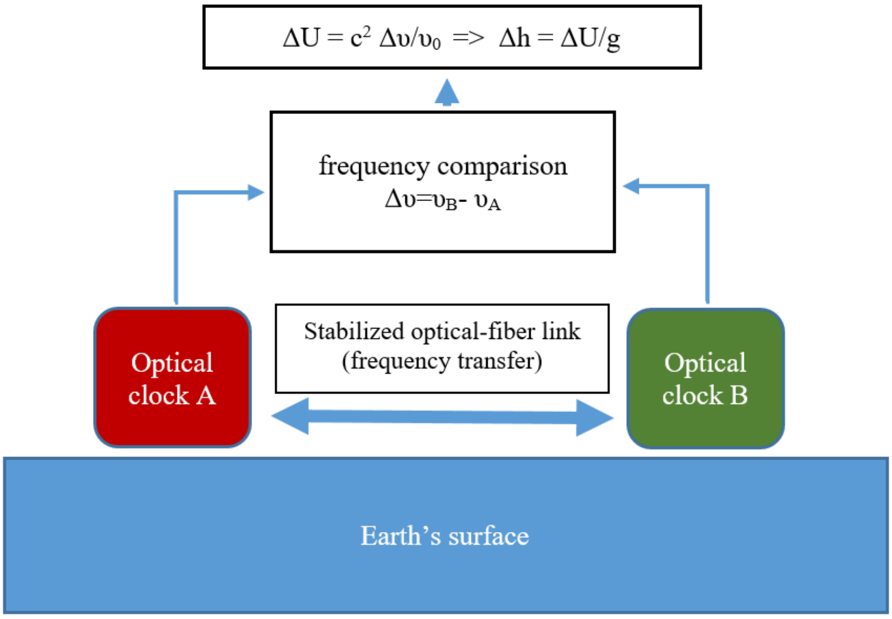

Figure 2. *Schematic of chronometric levelling between two observation sites*

Results

A key practical advantage of the method is that the accuracy of chronometric levelling is essentially independent of the distance between observation sites, in contrast to classical geometric levelling, whose error grows in proportion to the square root of the traverse length (Denker et al., 2018). This has been confirmed by a series of experimental studies: Takano et al. (2016) synchronously compared optical lattice clocks separated by 15 km within Tokyo, obtaining a geopotential difference corresponding to centimetre-level height accuracy (Takano et al., 2016). In 2020, the same group carried out a landmark experiment — comparing transportable optical clocks at the base and on the upper floor of the Tokyo Skytree tower, independently confirming the ≈ 450 m height difference by a purely relativistic method (Takamoto et al., 2020).

Chronometric levelling does not aim to replace classical methods but rather to complement them (Table 1). Its results are already comparable in accuracy to first-order levelling and to the GNSS/geoid approach (Mehlstäubler et al., 2018), yet they are obtained through a fundamentally different route, independent of a ground-based levelling network.

Table 1. *Comparison of height-determination methods*

Method	*Height accuracy*	*Distance dependence*	*Required infrastructure*
Geometric levelling	Millimetres–centimetres	Error grows with traverse length (√L)	Dense network of ground benchmarks
GNSS + geoid model	Centimetres–decimetres	Weak dependence on distance	Satellite infrastructure, geoid model
Chronometric levelling	Centimetres (and better)	Essentially independent of distance	Optical clock + frequency-transfer link

This sensitivity of classical, reference-surface-based height determination is well illustrated by ongoing work on national and regional height systems. A recent assessment of a possible transition of Ukraine's UCS-2000 coordinate system from the Krasovsky ellipsoid to GRS80 found that height-anomaly values would shift by roughly 0.5–1 m as a direct consequence of the change in ellipsoid parameters alone (Lukianchenko et al., 2025b), underscoring how strongly classical, ellipsoid-referenced height quantities depend on the choice of reference surface. Likewise, current efforts to construct and compare geoid and quasigeoid height maps across Europe highlight how strongly classical height determination still relies on the density and quality of the gravimetric data used to model the reference surface (Lukianchenko et al., 2025a). On top of that, chronometric levelling also shows good potential for use in geoid determination (Bondarescu et al., 2012) and even for studying the gravitational field of another celestial body, such as the Moon (Serant et al., 2026). Because chronometric levelling measures the gravitational potential directly — through the frequency of a clock — rather than through a geometric ellipsoid or an interpolated gravimetric surface, it is in principle free from precisely this class of reference-surface-dependent bias, which could make it a valuable independent check for national height-system and geoid modernization efforts of exactly this kind.

The path of relativistic geodesy from a theoretical proposal to a practical tool has taken more than four decades (Table 2).

Table 2. *Key milestones in the development of chronometric (relativistic) geodesy*

Year	Event / result
1975–1985	A. Bjerhammar formulates the concept of the relativistic geoid and the idea of determining the potential from clocks (Bjerhammar, 1985)
1983	M. Vermeer introduces the term "chronometric levelling" (Vermeer, 1983)
2010	First laboratory demonstration of a measurable gravitational redshift for a height difference of a few tens of centimetres (NIST) (Chou et al., 2010)
2016	Takano et al. — geopotential determination with synchronously linked optical clocks 15 km apart (Takano et al., 2016)
2017	Lion et al. — first quantitative contribution of clock comparisons to a high-resolution geopotential model (Lion et al., 2017)
2018	Review papers summarising methods and achievable accuracy of chronometric geodesy (Mehlstäubler et al., 2018; Denker et al., 2018)
2020	Takamoto et al. — verification of the ≈ 450 m height difference of the Tokyo Skytree with transportable optical clocks (Takamoto et al., 2020)

Conclusions

The unprecedented accuracy of modern optical atomic clocks has turned the gravitational redshift from a subject of fundamental tests of general relativity into a working geodetic tool. Chronometric levelling already demonstrates centimetre-level height accuracy today and, unlike classical methods, is essentially independent of the distance between observation sites. Continued development of satellite-borne optical clocks and global frequency-comparison networks, together with the ongoing modernization of national height systems and regional geoid models discussed above, is likely to make relativistic geodesy a fully-fledged component of future global height systems.

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