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Optimization of High-Precision Geometric Levelling Methodology for Monitoring Vertical Deformations of Building and Structure Foundations under Dynamic and Wartime Impacts

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SUMMARY

In the context of the full-scale war in Ukraine, there is a critically growing need for rapid and high-precision monitoring of the stability of foundations of buildings, bridges, and other engineering structures subjected to significant destructive impacts and dynamic loads. For a detailed analysis of the deformation characteristics of objects, high-precision geometric leveling is traditionally applied using methods for monitoring the stability of reference points. However, the use of modern digital levels with single-sided code staffs requires improving the measurement control methodology directly at the station. The purpose of this work is to assess how accounting for the magnitude of staff footing displacements affects the accuracy of elevation difference determination. The authors propose an alternative observation methodology using the digital level, which is based on changing the position of the footings of both staffs. The accuracy assessment was performed using the mathematical method of adjusting correlated measurements with the construction of covariance matrices and normal equations. Two scenarios for determining the zero shift were investigated, proving that complex leveling using two instrument horizons can be replaced by a simpler method involving changes in the position of the staff footings. The method of simultaneous displacement of both staff footings demonstrated the highest efficiency. To avoid a loss of accuracy in practice, the displacement of the footings must be recorded 3–4 times more precisely than the readings on the staff itself. The proposed approach optimizes the monitoring of war-induced destruction and structural deformations without compromising data reliability.

Keywords: High-precision leveling; Foundation deformations; War in Ukraine; Staff footing; Digital level; Accuracy assessment.

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Introduction

In the conditions of war in Ukraine, the necessity to monitor the stability of building foundations and engineering structures is significantly increasing. For this, high-precision leveling is used, in particular. Based on the results of such measurements, an evaluation of the deformation characteristics of engineering structures is conducted (Trevoho, I., 2023; Trevoho, I., 2021). When leveling using digital levels with one-sided coded rails, the issue of control and measurement precision at the station arises. To address this, it has been proposed to modify the position of the rail feet or the level's horizon. In this context, the question of assessing the precision of such digital leveling arises. Increasing the controllability of measurements with an evaluation of their precision is of great practical importance in the establishment of height networks.

Many scientific studies have been devoted to solving the problems of high-precision leveling (Baricevic, S., 2023; Burak, K., 2022; DSTU B V.2.1-30:2014, 2015; Fis, M., 2021; Zhytar, D., 2025; Kuchmister, J., 2020; Nalyvaiko, T., 2021; Oleskyv, R., 2023; Pimpi, J., 2022; Voytenko, S. P., 2023).

Unresolved issues include the absence of an assessment of the influence of changing the position of the heels on the accuracy of determining the elevation when using two rails. The task is to evaluate the influence of taking into account the magnitude of rail heel displacements on the precision of determining the elevation at the station during digital leveling.

Method and Theory

For high-precision leveling to control vertical movements of structures, optical-mechanical and digital levels were used, in particular the digital level (Fig. 1).

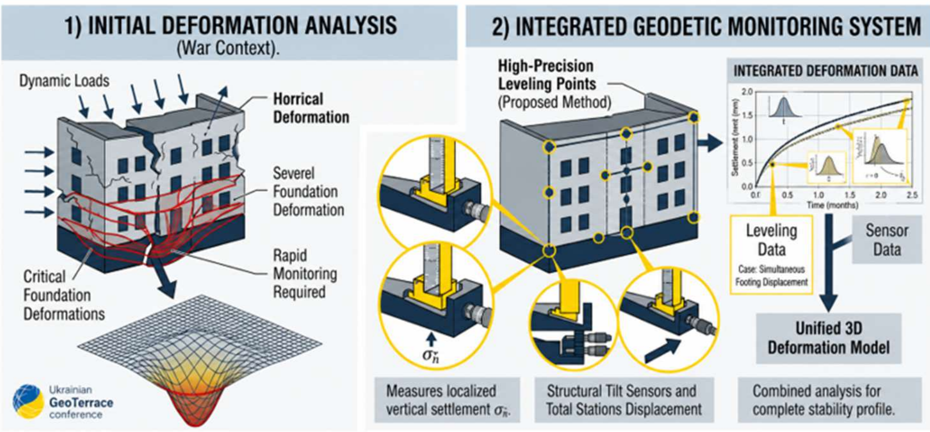

Figure 1. *Application of high-precision leveling to monitor the stability of foundations (under conditions of military influences)*

For high-precision leveling to control vertical movements of structures, optical-mechanical and digital levels were used, in particular the digital level.

The procedure for taking readings with a change in the position of the heels of the rails is presented in Table 1.

Table 1. *Observation scheme with altered rail heel positions*

Counts on the rail			
rear		**front**	
main scale	**addit/ scale**	**main scale**	**addit/ scale**
y_1 (1)	y_2 (2)	y_3 (3)	y_4 (4)
y_7^{**} (7)	y_8^{**} (8)	y_5^{*} (5)	y_6^{*} (6)

In Table 1 y*, y**- readings after shifting the heel of the front and rear rails, respectively. The order of taking readings is indicated in brackets. The magnitudes of the displacements are denoted as follows – $\mathbf{y_9}$ (for the rear rail), $\mathbf{y_{10}}$ (for the front rail).

A feature of this observation technique is the correlation of the exceedances that can be formed based on these readings

$$\mathbf{h_1} = \mathbf{y_1} - \mathbf{y_3};\ \mathbf{h_2} = \mathbf{y_2} - \mathbf{y_4};\ \mathbf{h_3} = \mathbf{y_1} - \mathbf{y_5}^* + \mathbf{y_{10}};\mathbf{h_4} = \mathbf{y_2} - \mathbf{y_6}^* + \mathbf{y_{10}};$$
$$\boldsymbol{h_5} = \boldsymbol{y_7}^{**} - \boldsymbol{y_5}^* - \boldsymbol{y_9} + \boldsymbol{y_{10}};\cdot \mathbf{h_6} = \boldsymbol{y_8}^{**} - \boldsymbol{y_6}^* - \boldsymbol{y_9} + \boldsymbol{y_{10}}. \tag{1}$$

The problem arises of equalizing multiple correlated measurements of the same quantity. In this case, the correction equations have the form

$$\boldsymbol{V} = \boldsymbol{A\delta h} + \boldsymbol{L}, \tag{2}$$

and the normal equation

$$\boldsymbol{R\delta h} + \boldsymbol{b} = \mathbf{0}, \tag{3}$$

where $\boldsymbol{R} = \boldsymbol{A^T P_h A},\ \boldsymbol{b} = \boldsymbol{A^T P_h L}.$

Let's consider the option where only the heel of the front rail moves.

In this case, the vectors of coefficients and free terms of the correction equations have the form

$$\boldsymbol{A} = \begin{pmatrix}\mathbf{1}\\\mathbf{1}\\\mathbf{1}\\\mathbf{1}\end{pmatrix}, \boldsymbol{L} = \begin{pmatrix}\mathbf{1}\\\mathbf{1}\\\mathbf{1}\\\mathbf{1}\end{pmatrix}, \boldsymbol{l_j} = \boldsymbol{h^0} - \boldsymbol{h_j}, \boldsymbol{h^0} = \boldsymbol{h_1}. \tag{4}$$

If we assume that all $\mathbf{y_i}$ are independent, then the covariance matrix $\mathbf{K_y}$ will be diagonal. Let us denote dispersion for readings on the rail as:

$$\boldsymbol{\sigma_{y_i}}^2 = \boldsymbol{\sigma_0}^2, \tag{5}$$

and for the heel displacement values as:

$$\boldsymbol{\sigma}^2 = \mathbf{c}\boldsymbol{\sigma_0}^2, \tag{6}$$

where $\boldsymbol{\sigma_0^2}$ is the variance of the error per unit weight.

Let us find the weight matrix $\mathbf{P_h}$. The covariance matrix $\mathbf{K_y}$ will have the vector (1,1,1,1,1,1,c) on the main diagonal. According to the generalized accuracy estimation theorem

$$\mathbf{K}_{\boldsymbol{h}} = \mathbf{B}\boldsymbol{K_y B^T}, \tag{7}$$

Let's consider two cases:

1) The magnitude of the zero offset is determined without error (c =0). Then the coefficient of the normal equation is

$$\boldsymbol{R} = \boldsymbol{A^T P_h A} = (\mathbf{1}\quad \mathbf{1}\quad \mathbf{1}\quad \mathbf{1})\boldsymbol{P_h}\begin{pmatrix}\mathbf{1}\\\mathbf{1}\\\mathbf{1}\\\mathbf{1}\end{pmatrix} = \textstyle\sum_{\boldsymbol{p}} = \frac{\mathbf{4}}{\mathbf{3}} \tag{8}$$

The free term of the normal equation

$$\boldsymbol{b = A^T P_h L = \left(\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}\right) \begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{pmatrix} = \frac{1}{3} \sum_{j=1}^{4} l_j} \tag{9}$$

Solution of the normal equation

$$\boldsymbol{\delta h = -\frac{b}{R} = -\frac{1}{4} \sum_{j=1}^{4} l_j.} \tag{10}$$

The adjusted excess value is the generalized arithmetic mean

$$\boldsymbol{\bar{h} = h^0 + \delta h = \frac{1}{4} \sum_{j=1}^{4} h_j} \tag{11}$$

with weight

$$\boldsymbol{P_{\bar{h}} = \sum_p = \frac{4}{3}.} \tag{12}$$

That is, theoretically, the excess determined by this method is determined with an accuracy of $\sigma_0 \sqrt{\frac{3}{4}}$. That is, the accuracy of this method is reduced by 25 % compared to the traditional method.

2) The magnitude of the zero offset is determined identically with the counts y_i. The weight matrix in this case will be as follows (c =1)

The solution to the normal equation will be as follows

$$\boldsymbol{\delta h = -\frac{b}{R} = -\frac{3}{8}(l_1 + l_2) - \frac{1}{8}(l_3 + l_4)} \tag{13}$$

The generalized weighted average will be

$$\boldsymbol{\bar{h} = h^0 - \frac{b}{R} = \frac{3}{8}(h_1 + h_2) + \frac{1}{8}(h_3 + h_4)} \tag{14}$$

and the weight

$$\boldsymbol{P_{\bar{h}} = \sum_p = \frac{8}{7}} \tag{15}$$

and the root mean square error

$$\boldsymbol{m_{\bar{h}} = \left(\frac{7}{24} V^T P_h V\right)^{\frac{1}{2}}.} \tag{16}$$

The latter has the following relationship with the mean square error of leveling at two horizons

$$\boldsymbol{m_{\bar{h}} = 1.323m,} \tag{17}$$

which means a reduction in accuracy by .32 %.

Conclusions

In the conditions of war in Ukraine, high-precision leveling is critically important for rapid monitoring and analysis of deformations of damaged structural foundations. A technologically simpler method of observation using the digital leveler is proposed, which is based on changing the position of the supports of both rails at the station instead of leveling using two horizons of the instrument. To prevent loss of measurement accuracy, the displacement of the rail supports must be determined 3–4 times more accurately than the indicators on the rails themselves.

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