GeoTerrace-2026-071

Predicting Land Subsidence over Underground Mining Excavations Using Approximate Mathematical Modelling: A Case Study of the Kalush-Golyn Deposit

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SUMMARY

The article examines the problems and methods of predicting man-made land surface subsidence in zones influenced by underground mining excavations in salt deposits. An analysis of existing approaches to assessing vertical land surface displacements is carried out, and the potential of using mathematical approximation models (linear, exponential, and second-degree polynomial) is considered. Based on long-term geodetic monitoring data for ground benchmarks along profile lines of the "Novo-Golyn" mine (2021–2024), calculations of expected subsidence values are performed, and predicted subsidence trough maps are constructed. A comparative analysis of the predicted values against actual measurement results from 2025 is conducted. It has been shown that the choice of the optimal approximation function depends on the stage of the geodynamic process: the exponential model is most effective in subsidence-attenuation zones, the polynomial model in areas with increasing deformation rates, and the linear model under conditions of uniform surface settlement. The obtained results help improve the reliability of short-term forecasting and minimize infrastructure risks in mining territories.

Keywords: Land surface subsidence; Geodetic monitoring; Mathematical modelling; Approximation functions; Underground excavations; Subsidence troughs.

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Introduction

Land surface subsidence is a widespread geological phenomenon of anthropogenic origin: according to an analysis of 290 cases in metropolitan areas worldwide, 76.92% of them are caused by human activity (Bagheri-Gavkosh et al., 2021; Shi et al., 2021). Intensive mineral resource extraction disrupts the natural state of the rock mass, inducing structural deterioration and gradual surface subsidence that can continue for decades after mining ceases.

This problem is particularly acute in salt deposits due to the plasticity and solubility of the rocks. A case in point is the salt deposit in Solotvyno, where the cessation of mining in 2010 resulted in sinkholes up to 200 m deep and a state of emergency, while satellite monitoring recorded a steady subsidence of over half a meter (Dobos et al., 2022). Similar processes are characteristic of the Kalush-Golyn potassium salt deposit, where deformations affect the urban area of Kalush (Kukhtar, Oleskiv, & Yavorska, 2025; Gera et al., 2025).

Timely detection and forecasting of these processes are critical for accident prevention and urban planning. Despite the introduction of remote sensing methods (radar interferometry (Lian et al., 2021), GNSS, and gravimetry), the reliability of short-term forecasting based on limited ground observation data remains an open question that requires further research in urbanized areas affected by mining excavation impacts.

Method and Theory

Ensuring a balance between mineral extraction and minimizing anthropogenic risks requires optimizing production processes, implementing effective real-time monitoring systems, and developing reliable predictive models to estimate subsidence rates and prevent accidents.

Extant literature suggests predicting subsidence through mathematical approximation models (Bo et al., 2024; Li et al., 2022), which describe vertical displacement distributions as a function of the time interval between geodetic surveys. Selecting the optimal model requires accounting for the process stage, rock geomechanical properties, geological and hydrogeological settings, and seasonal soil fluctuations.

In practice, the exponential function is most frequently used to approximate structural subsidence (Baran, 2012). Geometric models (linear or polynomial functions), trigonometric functions, and their combinations are used somewhat less frequently.

The **linear function** (best suited for modelling uniform subsidence that maintains a steady rate over the monitoring period) is defined by the following expression:

$$\boldsymbol{S_t = a_0 + a_1 t} \tag{1}$$

where ***St*** is the magnitude of subsidence (usually in mm) after a time interval ***t*** from the beginning of observations (the baseline epoch); $\boldsymbol{a_0}$ is the constant term characterizing the magnitude of subsidence at the time of the initial survey; $\boldsymbol{a_1}$ is a coefficient determined from the system of normal equations.

Among polynomial functions, **the quadratic parabolic model** is most commonly chosen:

$$\boldsymbol{S_t = a_0 + a_1 t + a_2 t^2} \tag{2}$$

Results

Mathematical approximation models describing the distribution of displacements as a function of the time interval between geodetic observations were used to forecast subsidence. Annual subsidence monitoring of ground benchmarks along profile lines within the "Novo-Golyn" salt mine (Ivano-Frankivsk Oblast) was continuously conducted from 2021 to 2024. Based on the recorded actual vertical displacements of the land surface and using approximation calculations, the projected subsidence values for the profile-line ground benchmarks were determined as of June 2025.

As a result of constructing the prognostic map of land surface subsidence troughs, the following zones were identified within which the largest land surface subsidence values are projected over the coming year.

The study area is located in the northeastern part of Kropyvnyk village, with a newly formed lake at its center. As shown in Fig. 1, the subsidence trough is expected to expand towards Kropyvnyk village. Specifically, the projected cumulative land subsidence recorded since the historical baseline (July 1968) exceeds 3,300 mm at the lake's edge as of June 2025. Meanwhile, for the applied approximation models, the zero-th epoch corresponds to the 2021 geodetic survey cycle. At the same time, the calculated annual subsidence forecast for the 2024–2025 period at the lake's edge is approximately 400 mm. In contrast, near benchmark Rp 267, located close to the farmstead nearest to the lake, the projected annual subsidence rate is about 72 mm.

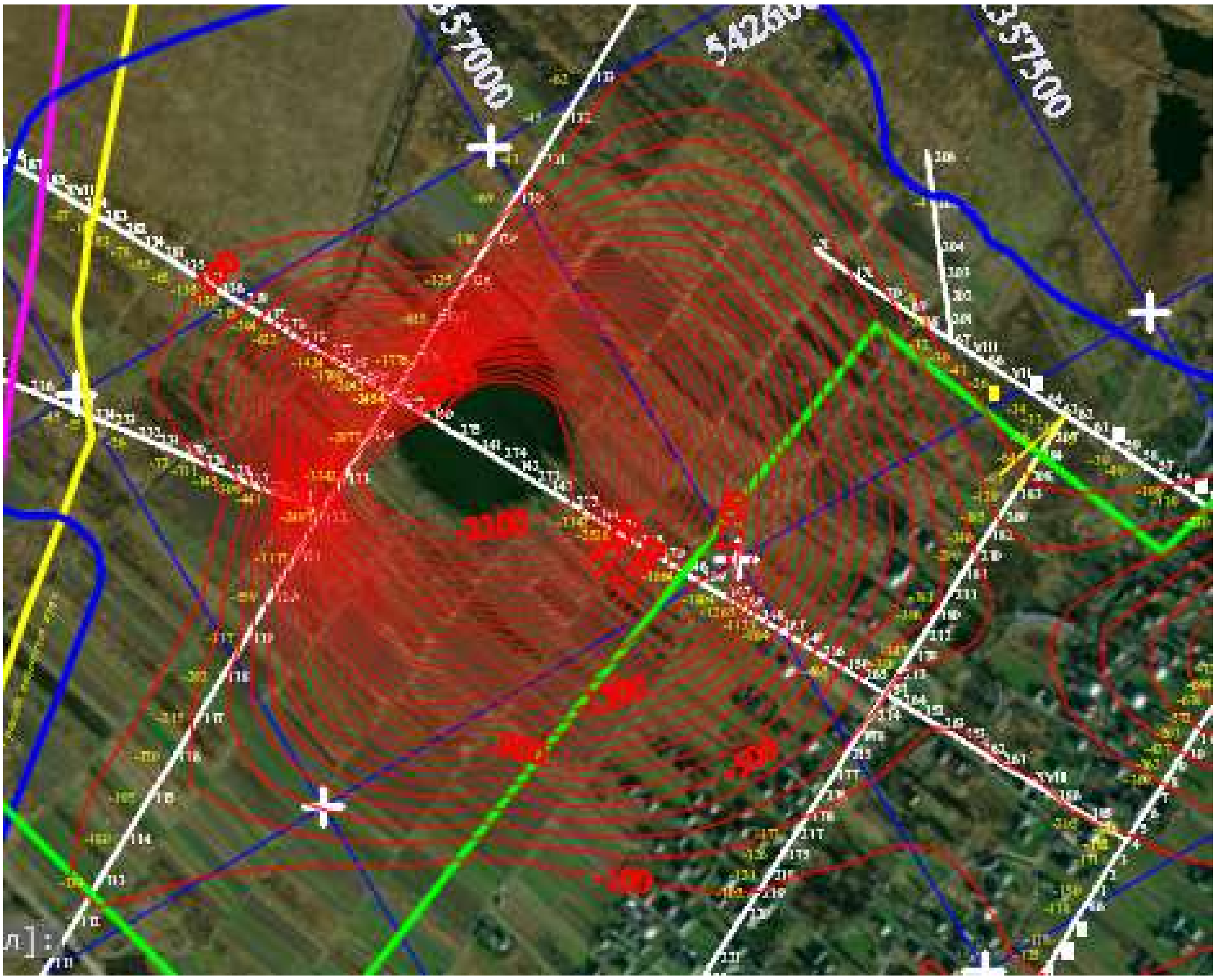

Figure 1. *Predicted map of land surface subsidence troughs (Part 1)*

Following the 2025 series of observations, a verification of the reliability of the approximation results was performed (Table 1).

The projected subsidence calculations for June 20, 2025, presented in the table, were performed using three approximation models based on actual data from the 2021–2024 period. For each approximation model, residual errors were calculated by comparing predicted subsidence with actual values from the 2025 measurements.

Table 1. *Determination of projected subsidence using the most common approximation functions based on observation results from 2021–2024*

Bench mark Name	Magnitude of actual subsidence as of June 20, 2025, mm	Mathematical approximation models					
		Linear		Exponential		Quadratic polynomial	
		Subsidence, mm	Residual error, mm	Subsidence, mm	Residual error, mm	Subsidence, mm	Residual error, mm
XVI	**-47**	-45	2	-45	2	-46	1
133	**-66**	-61	5	-61	5	-67	1
132	**-48**	-44	4	-44	4	-51	3
130	**-71**	-68	3	-67	4	-72	-1
128	**-318**	-321	3	-320	2	-323	-5
127	**-588**	-605	-17	-605	-17	-609	-20
126	**-1642**	-1747	-104	-1743	-101	-1770	-128
125	**-2274**	-2415	-142	-2410	-137	-2446	-172
124	**-2777**	-2927	-151	-2924	-147	-2957	-180
123	**-3160**	-3294	-135	-3292	-132	-3317	-158
122	**-2428**	-2462	-34	-2463	-35	-2462	-34
121	**-1097**	-1106	-9	-1106	-9	-1107	-10
120	**-543**	-553	-10	-553	-10	-554	-11
119	**-315**	-314	1	-314	1	-321	-6
118	**-296**	-290	6	-290	6	-297	-1
117	**-219**	-214	5	-214	5	-228	-9
116	**-174**	-168	6	-169	6	-179	-5
115	**-198**	-193	5	-194	4	-203	-5
114	**-183**	-178	5	-179	4	-188	-5
113	**-135**	-132	3	-133	2	-140	-5

From the Table 1, we see that in the center of the subsidence trough (benchmarks 123–126), the best results were obtained with the exponential model. Overall, a trend toward the attenuation of subsidence processes is observed in this part of the trough. In our opinion, the significant residuals are attributable to the use of only four available observation series for forecasting. Similar calculations were performed for all profile lines in the studied area, enabling a comparative analysis of the feasibility of applying a particular approximation model.

Conclusions

Based on the actual subsidence of ground benchmarks located within the underground mining zone, the projected annual subsidence values as of June 2025 were calculated using approximation functions. The calculated values were used to construct predicted subsidence troughs, which clearly illustrate the expansion trends of the subsidence zones and help assess the risks to local infrastructure and business facilities in these areas.

A comparative analysis was performed to evaluate the projected benchmark subsidence derived from various approximation functions against the actual values determined from the 2025 geodetic observations. The findings indicate that the exponential function yields better results in the subsidence attenuation zone. For areas experiencing an increasing rate of subsidence, polynomial approximation is recommended. The linear approximation function is best suited for uniform land surface subsidence.

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